LRFD

Section 1.1

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1.1.1 Design Philosophy

At present, design practice is based on one of three design philosophies: (1) allowable stress design, (2) load factor design, and (3) load and resistance factor design (LRFD). This section provides the information on the development of a reliability based design code such as AASHTO LRFD specification. The emphasis is placed on the definitions, formulas, the concept of limit states, statistical load models, and statistical resistance models needed for the code development. This section should be helpful to structural designers and should broaden their perspective by considering reliability based LRFD design as an important dimension of bridge design. However, a designer still can design bridges based on the LRFD codes without knowing structural reliability analysis background as described in this section.

1.1 Allowable Stress Design (ASD):

In ASD, it is ensured that the stresses in a structure under working or service loads do not exceed designated allowable values. The allowable values are obtained by dividing the yield stress or ultimate stress of the material by a factor of safety. The general format for an allowable stress design is:

$$\frac{R_n}{F.S.} \geq \sum_{i=1}^{m} Q_{ni} \qquad (1)$$

Where:

R_n = nominal resistance of the structural member expressed in units of stress

Q_{ni} = nominal working or service stresses computed under working loads due to load type i.

$F.S.$ = factor of safety

i = type of load (i.e., dead load, live load, wind load, etc.)

m = number of load types

The left-hand side of Equation (1) represents the allowable stress of the structural member or component under a given loading condition (e.g., tension, compression, bending, or shear). The right-hand side of the equation represents the combined stress produced by various load combinations (e.g., dead, live, or wind load). One should realize that in allowable stress design, the factor of safety is applied only to the resistance term, and safety is evaluated at the service load. Thus, ASD is characterized by the use of unfactored “working” loads in conjunction with a single factor of safety applied to the resistance. Because of the greater variability and unpredictability of the live load and other loads in comparison with the dead load, a uniform reliability is not possible with ASD.

1.2 Load Factor Design (LFD):

In LFD (with load factors), it is ensured that factored load combinations do not exceed the maximum required strength of the structure or component.

It takes the form:

$$R_n \geq \sum_{i=1}^{m} \gamma_i Q_{ni} \qquad (2)$$

Where:

R_n = nominal required strength of the member (such as plastic moment strength)

Q_n = nominal load effect (e.g., axial force, shear force, bending moment)

γ_i = load factor (For example: 1.3 for dead load and 2.17 for live load)

i = type of load (*D* = dead load, *L* = live load, *W* = wind load, etc.)

m = number of load types

Note that in this method, safety is incorporated only in the load term and is evaluated at the required limit state. Applying a factor of safety to the load term is more appropriate than ASD because uncertainty associated with loads is higher than that associated with resistances. A uniform reliability cannot be fully achieved with LFD because only factors of safety (here called load factors) are applied to loads.

1.3 Load and Resistance Factor Design (LRFD):

In LRFD, it is ensured that the factored load effects do not exceed the factored nominal resistance of the structural member or component. There are two safety factors: one applied to the loads, the other to the resistance. This is a more rational approach because both the loads and the resistances have different uncertainties. Thus, LRFD takes the form:

$$\phi\, R_n \geq \sum_{i=1}^{m} \gamma_i Q_{ni} \tag{3}$$

Where:

R_n = nominal resistance of the structural member

Q_n = nominal load effect (e.g., axial force, shear force, bending moment)

ϕ = resistance factor ($\leq$ 1.0) (e.g., 0.9 for beams, 0.85 for columns)

γ_i = load factor (usually > 1.0) corresponding to Q_{ni} (e.g., 1.25*D* + 1.75 (L+I))

i = type of load (e.g., *D* = dead load, *L* = live load)

m = number of load type

LRFD uses separate factors for each load and can therefore reflect the degree of uncertainty of different loads and combination of loads. As a result, more uniform reliability can be achieved. Uniform reliability means that individual structural members have the same probability of safety. For bridges, the probability of safety of a member can be evaluated through reliability analysis. Basically, the reliability analysis can be achieved by two techniques. The first technique is called reliability (safety) index approach and the other is called Monte Carlo Simulation.

In the current LRFD specifications, the resistance factors were developed mainly through a calibration in order to reach the target safety index, β, of 3.5

In order to develop load and resistance factors for the new LRFD bridge codes, the work involved several steps: the development of statistical load models, statistical resistance model, reliability analysis procedure, selection of the target reliability index and calibration of the load and resistance factors for the code. A brief description of the individual steps are described in Sections 1.1.6 through 1.1.9.

The calibration of the load and resistance factors as described in Section 1.1.9 can not be done without first completing the previous tasks described in Sections 1.1.6 through 1.1.8. Section 1.1.2 describes the fundamental probability theory. Section 1.1.3 though 1.1.5 provide very important background to be used for the development of statistically load and resistance models and description of reliability analysis procedures.

1.1.2 Fundamental Probability Theory

2.1 Definitions

Sample Space – considering an experiment such as concrete cylinder test for measuring concrete ultimate stress, f'c, all the possible outcomes comprise a sample space. Outcome of each trial is called a realization or a Sample Functions.

Event – a range of outcomes is defined as an event.

P( ) – The notation $P(\ )$ represents a probability function. If E represents an event, and Ω represents a sample space, then $P(E) =$ probability of event E and $P(\Omega) =$ the probability of an event corresponding to the entire sample space. Therefore, $0 \le P(E) \le 1$ and $P(\Omega) = 1$.

Mutually exclusive events – Two or more mutually exclusive events cannot occur simultaneously. For n mutually exclusive events $E_1, E_2, \ldots. E_N$,
$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i)$ where $P\left(\bigcup_{i=1}^{n} E_i\right)$ represents the probability of occurrence of E_1 or E_2 or …or E_n. For example, define 3 mutually exclusive events

$$E_1 = \{0 \le f'c < 2000\,psi\};$$
$$E_2 = \{2000 \le f'c < 5000\,psi\};\ and$$
$$E_3 = \{f'c \ge 5000\,psi\}$$

The union of these 3 mutually exclusive events is the entire sample space, i.e. $\bigcup_{i=1}^{3} E_i = \{0 \le f'c < \infty\}$

Random Variables – A random variable is defined as a function that maps events onto intervals on the axis of real numbers. For example;

$$X(f'c) = \begin{Bmatrix} 1\ \ if\ \ E_1 = 0 \le f'c < 2000\,psi \\ 2\ \ if\ \ E_2 = 2000\,psi \le f'c < 5000\,psi \\ 3\ \ if\ \ E_3 = f'c \ge 5000\,psi \end{Bmatrix} \qquad (1)$$

So the random variable x can have three discrete integer values (i.e. 1, 2, and 3). Then x is called a discrete random variable. A random variable can be a continuous random variable. For example, let $X(f'c) = f'c$, then an event $f'c = 3500\,psi$ corresponds to the random variable $X(f'c) = 3500\,psi$.

Probability Mass Function (PMF) $P_X(\chi)$ – Let $P_X(x)$ = probability mass function that a discrete random variable X is equal to a specific value x. Therefore, $p_X(x) = P(X = x)$. For example, let X be a discrete random variable representing concrete strength $f'c$ as defined in equation (1). Assume the values of the probability mass function $P_X(x)$ as:

$$P_X(1) = 0.05$$
$$P_X(2) = 0.85$$
$$P_X(3) = 0.1$$

Figure 1.1.2.1 shows these three values of the probability mass function $P_X(x)$.

Cumulative distribution Function (CDF) $F_X(\chi)$ – The total sum of all probability functions corresponding to values less than or equal to x. $F_X(x) = P(X \le x)$ as shown in Figure 1.1.2.2. $F_X(x)$ is an increasing function of x.

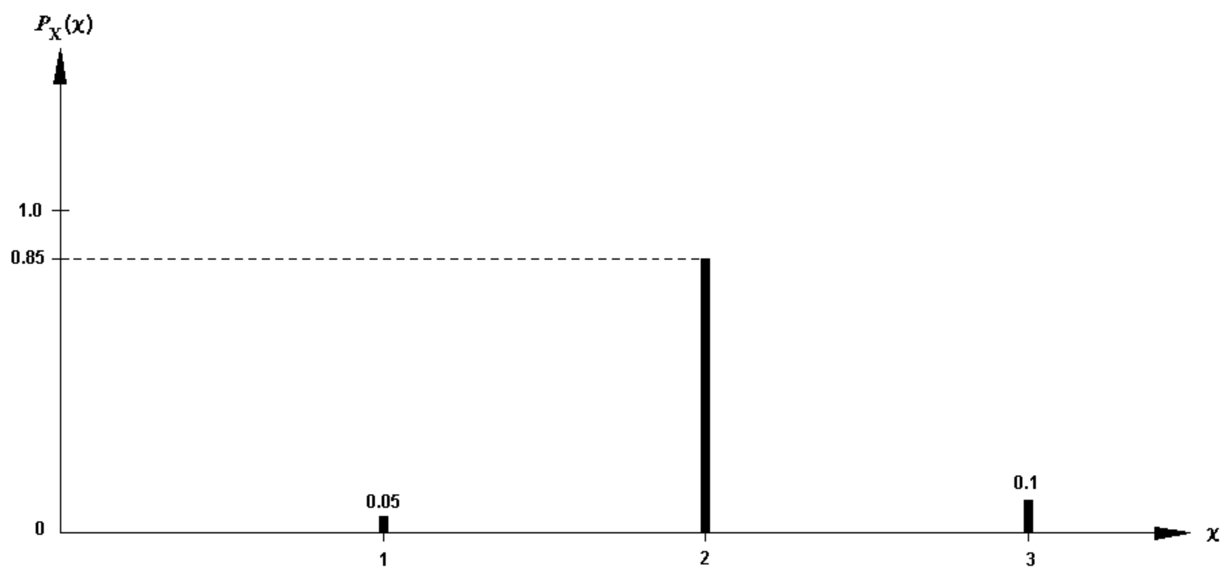


Figure 1.1.2.1 Probability Mass Functions

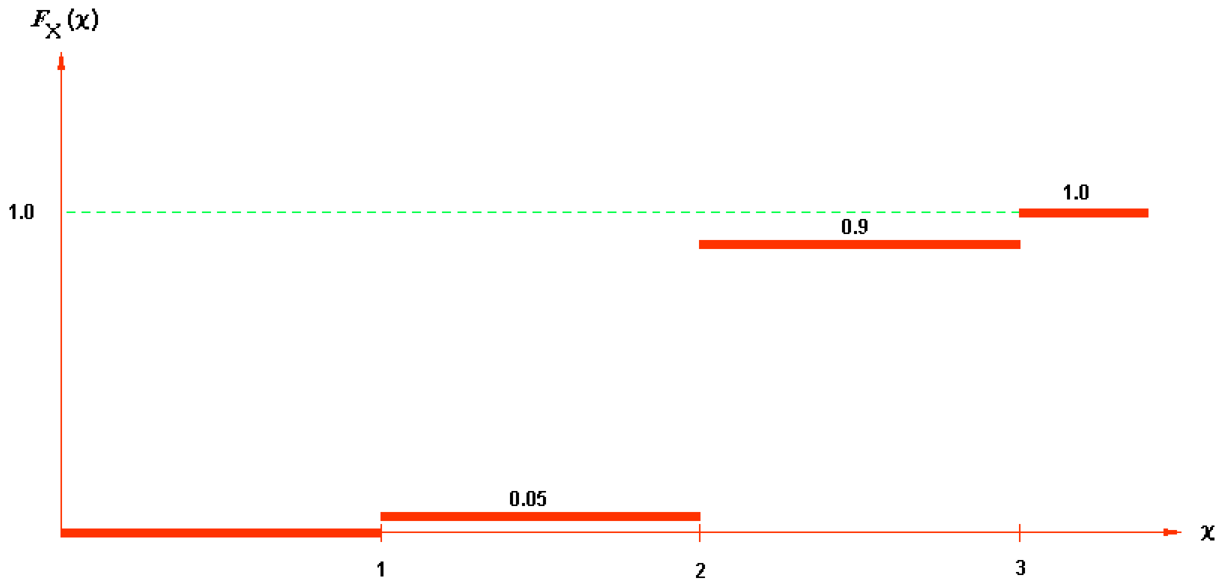


Figure 1.1.2.2 Cumulative Distribution Function

Probability Density Function (PDF) $f_X(\chi)$ – $f_X(x) =$ Probability functions in which a continuous random variable X is equal to a specific value x. In other words a, probability density function is the first derivative of the cumulative distributions function.

$$f_X(x) = \frac{d}{dx} F_X(x) \tag{2}$$

$$F_X(x) = \int_{-\infty}^{x} f_X(\rho) d\rho \tag{3}$$

Figures 1.1.2.3 (a) and (b) represent typical PDF and CDF, respectively.

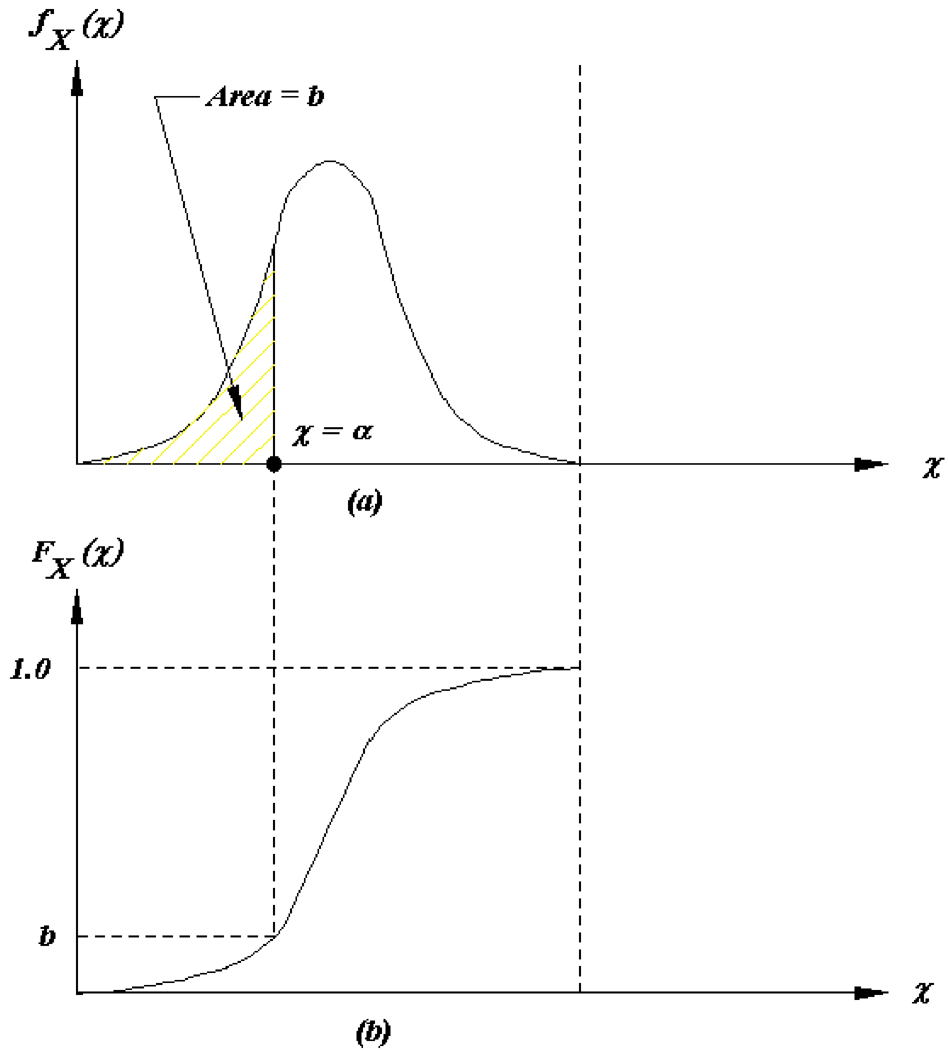


Figure 1.1.2.3: (a) PDF and (b) CDF.

Equation (3) represents the shaded area under PDF as shown in Figure 1.1.2.3 (a) when $x = a$.

2.2 Statistic Parameters

The purpose of using the following statistic parameters is to describe the properties of any random variable.

Mean value of a random variable X denoted as μ_X:

$$\mu_X = \int_{-\infty}^{\infty} x f_X(x)dx \qquad \text{for a continuous random variable}$$

$$= \sum x_i P_X(x_i) \qquad \text{for a discrete random variable}$$

The above equations for μ_X requires knowing the PDF of a particular random variable, X. For a set of test data $\{x_1, x_2 ... x_n\}$, the mean μ_X can be estimated by

$$\mu_X = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{4}$$

Expected value of X denoted by $E(X)$: $E(X) = \mu_X$

Expected value of X^n denoted by $E(X^n)$:

$$E(X^n) = \int_{-\infty}^{+\infty} x^n f_X(x)dx$$

$E(X^n)$ is also called the nth moment of X

Variance of X denoted by σ_X^2:

$$\sigma_X^2 = E(X-\mu_X)^2 = \int_{-\infty}^{+\infty}(x-\mu_X)^2 f_X(x)dx$$

The relationship among the mean, variance and second moment of a random variable X is

$$\sigma_X^2 = E(X^2) - \mu_X^2$$

Standard deviation of X denoted by σ_X:

$$\sigma_X = \sqrt{\sigma_X^2}$$

For a set of test data, $\{x_1, x_2 ... x_n\}$, the standard deviation can be estimated by

$$\sigma_X = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - n(\bar{x})^2}{n-1}} \tag{5}$$

Coefficient of variation denoted by V_X:

$$V_X = \frac{\sigma_X}{\mu_X}$$

Standard form of X denoted by Z:

$$Z = \frac{X-\mu_X}{\sigma_X}$$

$$\mu_Z = E(Z) = E\left(\frac{X-\mu_X}{\sigma_X}\right) = \frac{1}{\sigma_X}(E(X) - E(\mu_X))$$

$$= \frac{1}{\sigma_X}(\mu_X - \mu_X) = 0$$

$$\sigma_Z^2 = E(Z^2) - \mu_Z^2 = E\left[\left(\frac{X - \mu_X}{\sigma_X}\right)^2\right] - 0$$

$$= \frac{1}{\sigma_X^2} E\left[(X - \mu_X)^2\right] = \frac{\sigma_X^2}{\sigma_X^2} = 1$$

Therefore the mean of the standard form of a random variable is 0 and its variance is 1.

Conditional Probability:

Given two events E_1 and E_2, the conditional probability of E_1 occurring if E_2 has already occurred is defined as

$$P(E_1|E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} \tag{1}$$

The symbol $"\cap"$ is called "intersection" and means that events E_1 and E_2 occur simultaneously. If two events are statistically independent, then the occurrence of one event has no effect on the other event, then equation (1) reduces to

$$P(E_1|E_2) = P(E_1) \ and \ P(E_2|E_1) = P(E_2).$$

2.3 Common Random Variables

Uniform Random Variables:

A uniform random variable has all numbers that are equally likely to appear. Therefore, the PDF has a constant value for all possible values of the random variable with a range $[a,b]$.

$$PDF = f_X(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & otherwise \end{cases}$$

The mean and variance are

$$\mu_x = \frac{a+b}{2}; \sigma_X^2 = \frac{(b-a)^2}{12}$$

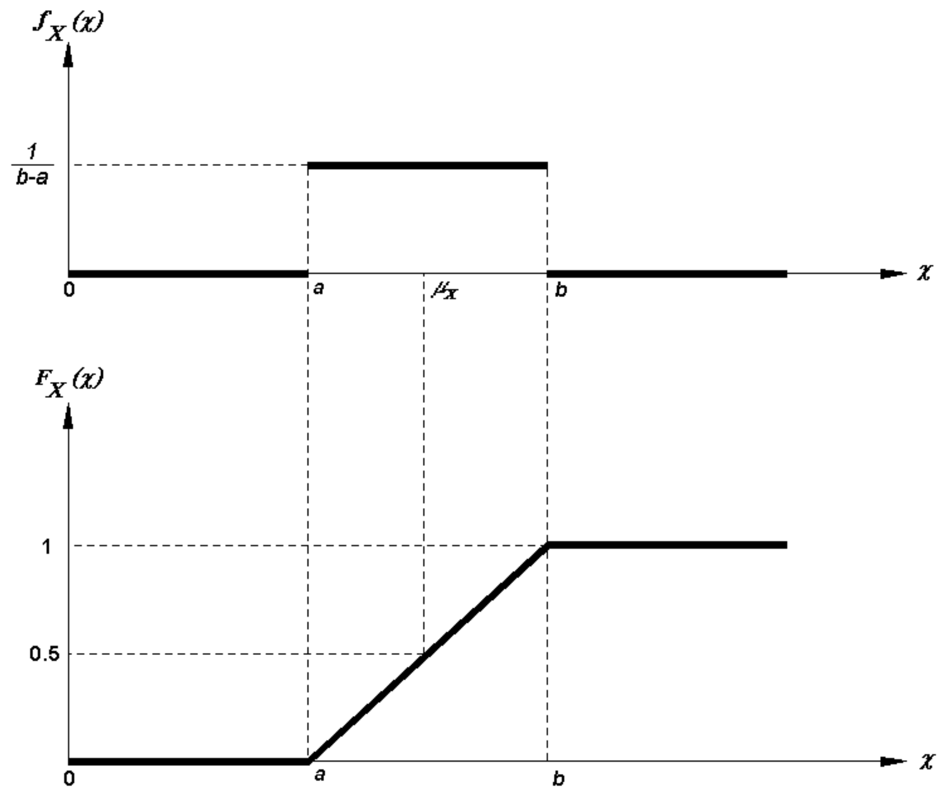


Figure 1.1.2.4: Uniform Random Variable – PDF and CDF

Normal random variables:

The PDF of a normal random variable X is

$$f_X(x) = \frac{1}{\sigma_X\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu_X}{\sigma_X}\right)^2\right]$$

The PDF and CDF of a normal random variable are shown in Figure 1.1.2.5.

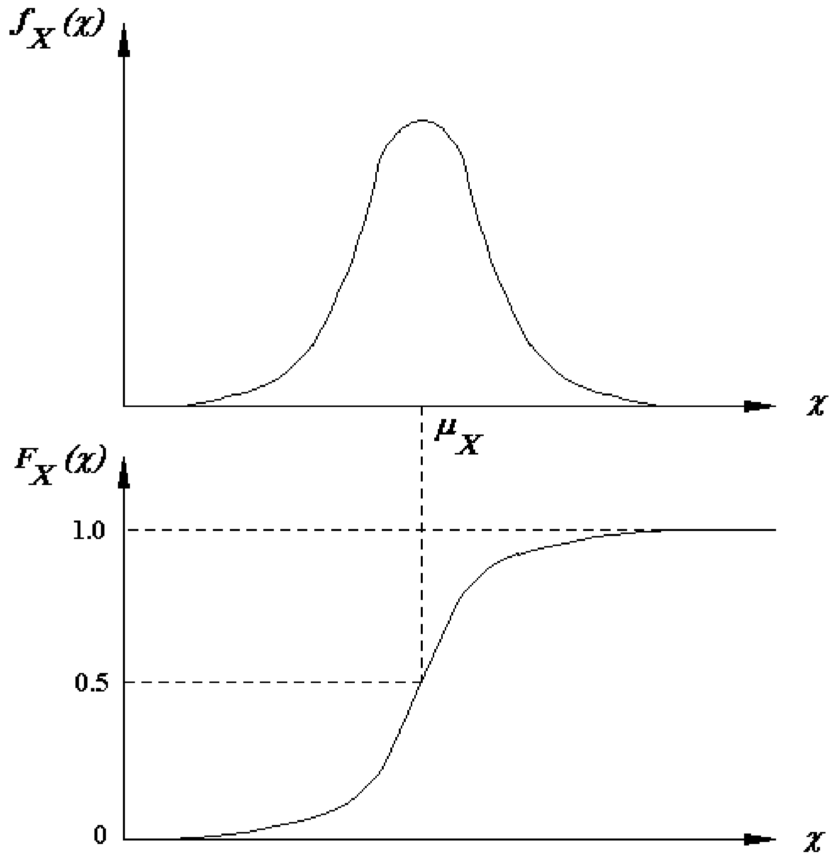


Figure 1.1.2.5 PDF and CDF of a normal random variable.

Standard normal random variable Z ***:***
The PDF of a standard normal random variable Z is

$$\phi(z) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(z)^2 \right] = f_Z(z)$$

The CDF of the standard normal random variable is typically denoted by $\Phi(z)$

The PDF and CDF of a standard normal random variable are shown in Figure 1.1.2.6. From Section 1.1.2.2, $Z = \frac{X - \mu_X}{\sigma_X}$, therefore

$$X = \mu_X + Z\sigma_X$$

$$F_X(x) = P(X \le x) = P(\mu_X + Z\sigma_X \le x) = P\left(Z \le \frac{x-\mu_X}{\sigma_X}\right) = P(Z \le z)$$

or $F_X(x) = F_Z(z) = \Phi\left(\frac{x-\mu_X}{\sigma_X}\right)$ (1)

Equation (1) means that the standard normal random variable, Z, can be used to obtained the CDF of an arbitrary normal random variable, X.

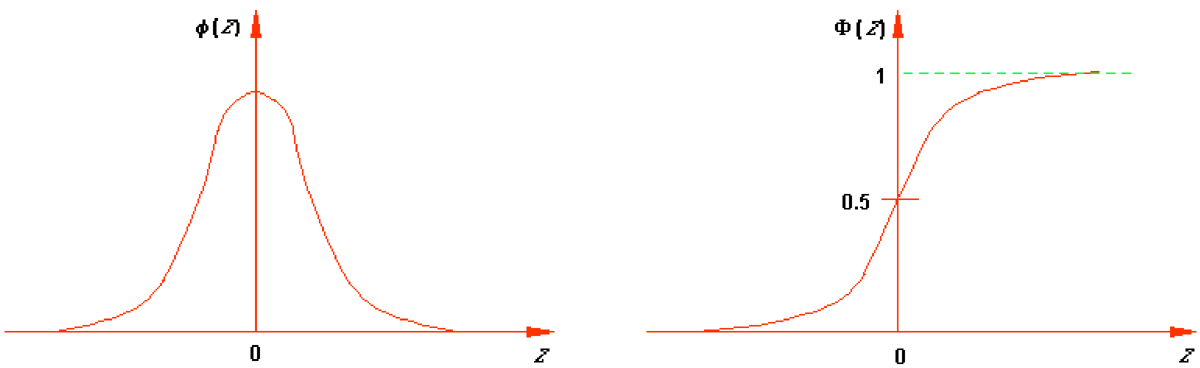


Figure 1.1.2.6 PDF and CDF of a standard normal random variable

Lognormal random variable:

A random variable X is lognormal if

$Y = \ln(X)$ is a normal random variable.

$$F_X(x) = P(X \le x) = P(\ln X \le \ln x) = P(Y \le y) = F_Y(y)$$

$F_X(x) = F_Y(y) = \Phi\left(\frac{y-\mu_Y}{\sigma_Y}\right)$ where $y = \ln(x)$;

$\mu_Y = \mu_{\ln(X)} =$ mean value of $\ln(X)$

$$\sigma^2_{\ln(X)} = \ln(V_X^2 + 1)$$

$$\mu_{\ln(X)} = \ln(\mu_X) - \frac{1}{2}\sigma^2_{\ln(X)}$$

If V_X is less than 0.2, then

$$\sigma^2_{\ln(X)} \approx V_X^2$$

$$\mu_{\ln(X)} \approx \ln(\mu_X)$$

The PDF of a lognormal random variable is shown in Figure 1.1.2.7.

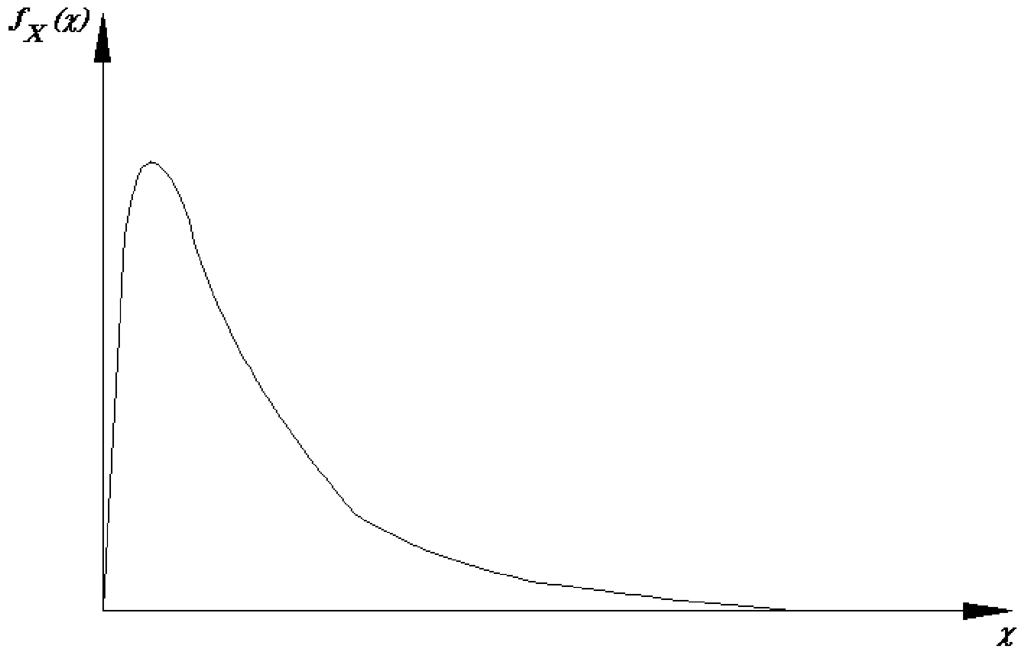


Figure 1.1.2.7 PDF of a lognormal random variable.

Random Vector:
A set of random variables is called a random vector. For example $\{X_1, X_2, X_3 ... X_n\}$ is a random vector with random variables of $X_1, X_2, ... X_n$. Similar to the CDF and PDF of a random variable, the CDF and PDF of a random vector are called "joint cumulative distribution function" and "joint probability density function", respectively. The joint cumulative distribution function is defined as

$$F_{X_1 X_2 ..... X_n}(x_1, x_2, ... x_n) = P(X_1 \le x_1, X_2 \le x_2, ..., X_n \le x_n)$$
$$= P(X_1 \le x_1 \cap X_2 \le x_2 \cap ... \cap X_n \le x_n) \qquad (1)$$

The joint probability density function is defined as

$$f_{X_1 X_2 .... X_n}(x_1, x_2, ... x_n) = \frac{\partial^n F}{\partial x_1, x_2 ... x_n}(x_1, x_2, ... x_n) \qquad (2)$$

For a random vector $\{X, Y\}$,

$$f_{XY}(x, y) = \frac{\partial^2 F_{XY}}{\partial x \partial y}(x, y)$$

and $f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x, y) dy$

$$f_Y(x) = \int_{-\infty}^{\infty} f_{XY}(x, y) dx$$

where $f_X(x)$ and $f_Y(y)$ are called marginal probability density functions.

Also, the conditional distribution function for the random vector $\{X, Y\}$ is

$$f_X(x|y) = \frac{f_{XY}(x, y)}{f_Y(y)} \tag{3}$$

If the random variables X and Y are statistically independent as described in Section 1.1.2.2, equation (3) reduces to

$$f_{XY}(x, y) = f_X(x) f_Y(y) \tag{4}$$

Covariance of Random Vectors:

The covariance of two random variables X and Y is defined as

$$COV(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x - \mu_X)(y - \mu_Y) f_{XY}(x, y)dxdy \tag{5}$$

Coefficient of Correlation:

The coefficient of correlation between two random variables X and Y is defined as

$$\rho_{XY} = \frac{COV(X, Y)}{\sigma_X \sigma_Y} \tag{6}$$

and $-1 \le \rho_{XY} \le 1$

ρ_{XY} shows the degrees of "linear" dependence between the two random variables X and Y. When $\rho_{XY} = 0,$ it means that the random variables X and Y are not linearly related to each other. However, it doesn't mean that X and Y are statistically independent because X and Y may have a nonlinear relationship to each other.

2.4 Normal Probability Paper

Normal probability paper can be constructed by redefining the vertical scale of the normal CDF so that the normal CDF will plot as a straight line. Hence, the values on the vertical axis of a normal probability paper are not evenly spaced as shown in Figure 1.1.2.8.

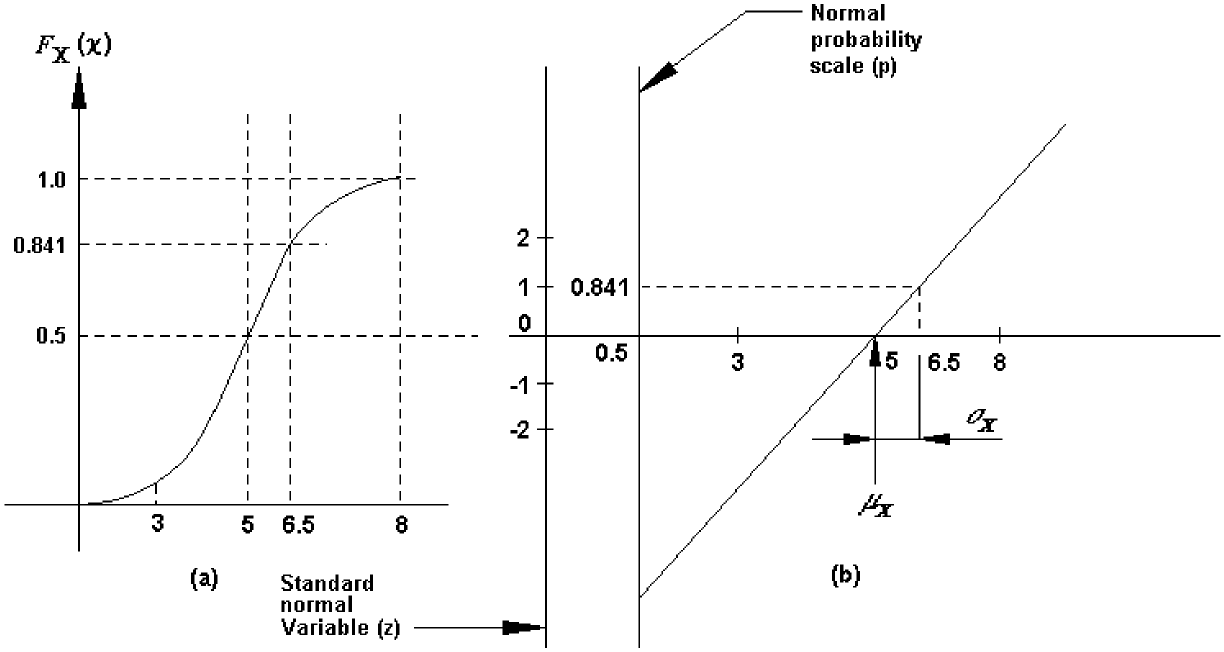


Figure 1.1.2.8 Construction of normal probability paper. (a) normal CDF (b) probability paper

In Section 1.1.2.2, the standard form of a normal random variable X is

$$Z = \frac{X-\mu_X}{\sigma_X} = \left(\frac{1}{\sigma_X}\right)X + \left(\frac{-\mu_X}{\sigma_X}\right) \tag{1}$$

for any specific value of Z, z,

$$z = \left(\frac{1}{\sigma_X}\right)x + \left(\frac{-\mu_X}{\sigma_X}\right) \tag{2}$$

From Equation (1) in Section 1.1.2.3, the normal CDF can be expressed as

$$F_X(x) = p = \Phi\left(\frac{x-\mu_X}{\sigma_X}\right) \tag{3}$$

Inversely, Equation (3) leads to

$$\Phi^{-1}(p) = \frac{x-\mu_X}{\sigma_X} = \left(\frac{1}{\sigma_X}\right)x - \left(\frac{\mu_X}{\sigma_X}\right) \quad (4)$$

$$z = \Phi^{-1}(p) = \left(\frac{1}{\sigma_X}\right)x - \left(\frac{\mu_X}{\sigma_X}\right) \quad (5)$$

By using z as the vertical axis in Figure 1.1.2.8 (b), it can be seen that the values on this scale are evenly spaced. The following formula can be used for evaluate z :

$$z = \Phi^{-1}(p) = -t + \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} \text{ for } p \le 0.5$$

where

$$c_0 = 2.515517;\ c_1 = 0.802853;\ c_2 = 0.010328,$$
$$d_1 = 1.432788;\ d_2 = 0.189269;\ d_3 = 0.001308,\ t = \sqrt{-\ln(p^2)}$$

for $p > 0.5; z = -\Phi^{-1}(p^*)$ in which $p^* = (1-p)$.

From experimental test results, the normal probability paper can be generated by the following steps:

Let the data plotted include n test results: $x_1,\ x_{2,}\ ...x_n$. It is assumed the values of $x_1,....x_n$ are arranged in an increasing order. Then, the first test result is plotted at the intersection of x_1 on the horizontal scale and probability $p_1 = \frac{1}{(n+1)}$ on the vertical scale. The ith test result is plotted at the intersection of x_i and the probability, $p_i = \frac{i}{(n+1)}$. It is convenient to replace p_i by the standard normal variable z_i using $z_i = \Phi^{-1}(p_i)$ as described in Equation (5).

Example: Consider a random variable, X , representing concrete cylinder test results. Total number of test data (in terms of ksi) is *9*. They are *4.6, 4.9, 5.0, 5.1, 5.1, 5.2, 5.2, 5.3, 5.5.* Use probability paper to evaluate the data statistically.

Solution: Find the probability, p_i , and standard normal variable, $z_{i,}$ of test data x_i :

Table 1.1.2.1

i	x_i	$p_i = \frac{i}{n+1}$	$z_i = \Phi^{-1}(p_i)$
1	4.6	0.1	-1.282
2	4.9	0.2	-0.842
3	5.0	0.3	-0.524
4	5.1	0.4	-0.253
5	5.1	0.5	0
6	5.2	0.6	0.253
7	5.2	0.7	0.524
8	5.3	0.8	0.842
9	5.5	0.9	1.282

From Table 1.1.2.1 the probability paper is generated and shown in Figure 1.1.2.9.

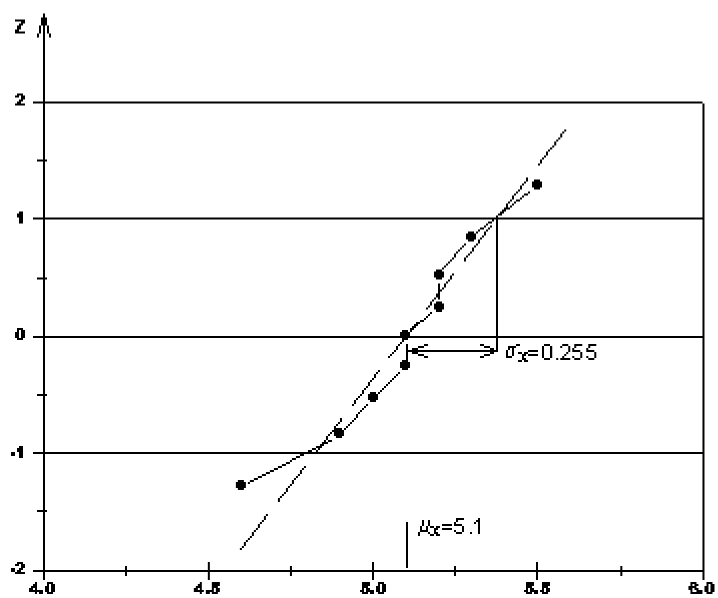


Figure 1.1.2.9 Probability Paper

For comparison, equations (4) and (5) in Section 1.1.2.2 are used to calculate the sample statistics μ_X and σ_X. They are

$$\mu_X = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{9}(4.6 + 4.9 + 5.0 + 2*5.1 + 2*5.2 + 5.3 + 5.5) = 5.1$$

$$\sigma_X = \sqrt{\frac{\left(\sum_{i=1}^{n} x_i^2\right) - n(\bar{x})^2}{n-1}} = \sqrt{\frac{234.61 - 9*(5.1)^2}{9-1}} = 0.255$$

From Figure 1.1.2.9, it can be seen that the data appear to follow a straight line. Therefore the test results follow a normal distribution. The dash line in the figure is plotted based on the μ_X and σ_X calculated above.

1.1.3 Applications of Random Variables

3.1 Linear Functions of Random Variables

If a random variable, Y, is a linear function of random variables $X_1,\ X_2,...X_n$;

$$Y = a_0 + a_1 X_1 + a_2 X_2 + ... + a_n X_n = a_0 + \sum_{i=1}^{n} a_i X_i,$$

then the mean of Y is

$$\mu_Y = a_0 + \sum_{i=1}^{n} a_i \mu_{X_i} \tag{1}$$

and the variance of Y is

$$\sigma_Y^2 = E\left[\left(Y - \mu_Y\right)^2\right]$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j \rho_{X_i X_j} \sigma_{X_i} \sigma_{X_j} \tag{2}$$

If $X_1,\ X_2,...X_n$ are uncorrelated with each other, then $\rho_{X_i X_j} = 0$ for $i \neq j$ and equation (2) reduces to

$$\sigma_Y^2 = \sum_{i=1}^{n} a_i^2 \sigma_{X_i}^2 \tag{3}$$

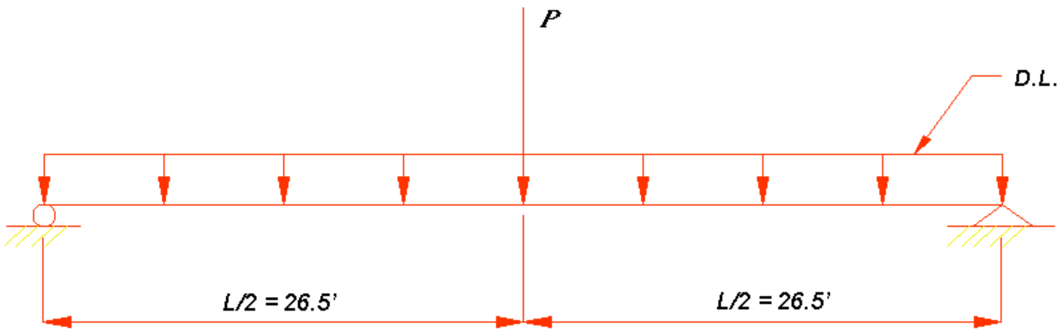
Example 1.1.3.1: A simply supported prestressed I-Girder is loaded with dead load (self weight + slab weight), DL, and a concentrated dead load (steel diaphragm) at center of the beam, P. The allowable moment capacity at the mid-span of the girder is M_R. Girder span length is 53 feet. All three random variables of DL, P, and M_R are uncorrelated normal random variables with statistical parameters shown as follows:

$$\mu_{DL} = 1.6 k/feet;\ V_{DL} = 10\%$$

$$\mu_P = 0.3 k/feet;\ V_P = 15\%$$

$$\mu_{M_R} = 1200\ \ k - feet;\ V_{M_R} = 12\%$$

what is the probability of failure of the girder?



Sol: Calculate the standard deviation of DL, P, and M_R

$$\sigma_{DL} = \mu_{DL} \cdot V_{DL} = 1.6 * 0.1 = 0.16(k/feet)$$

$$\sigma_P = \mu_P \cdot V_P = 0.3 * 0.15 = 0.045(kips)$$

$$\sigma_{M_R} = \mu_{M_R} \cdot V_{M_R} = 1200 * 0.12 = 144(k - feet)$$

The demand moment at the center of the girder is

$$M = \frac{(DL)(L)^2}{8} + \frac{PL}{4} = 351.12(DL) + 13.25(P)$$

Let $Y = M_R - M = M_R - 351.12(DL) - 13.25(P)$ (a)

Equation (a) shows that Y is a linear function of random variables M_R, DL, and P.

Therefore:

$$\mu_Y = \sum_{i=1}^{3} a_i \mu_{X_i}$$

$$= \mu_{M_R} - 351.12\mu_{DL} - 13.25\mu_P$$

$$= 1200 - (351.12)(1.6) - (13.25)(0.3) = 634.23(k - feet)$$

$$\sigma_Y^2 = \sum_{i=1}^{3} a_i^2 \sigma_{X_i}^2$$

$$= (1)^2(144)^2 + (351.12)^2(0.16)^2 + (13.25)^2(0.045)^2$$

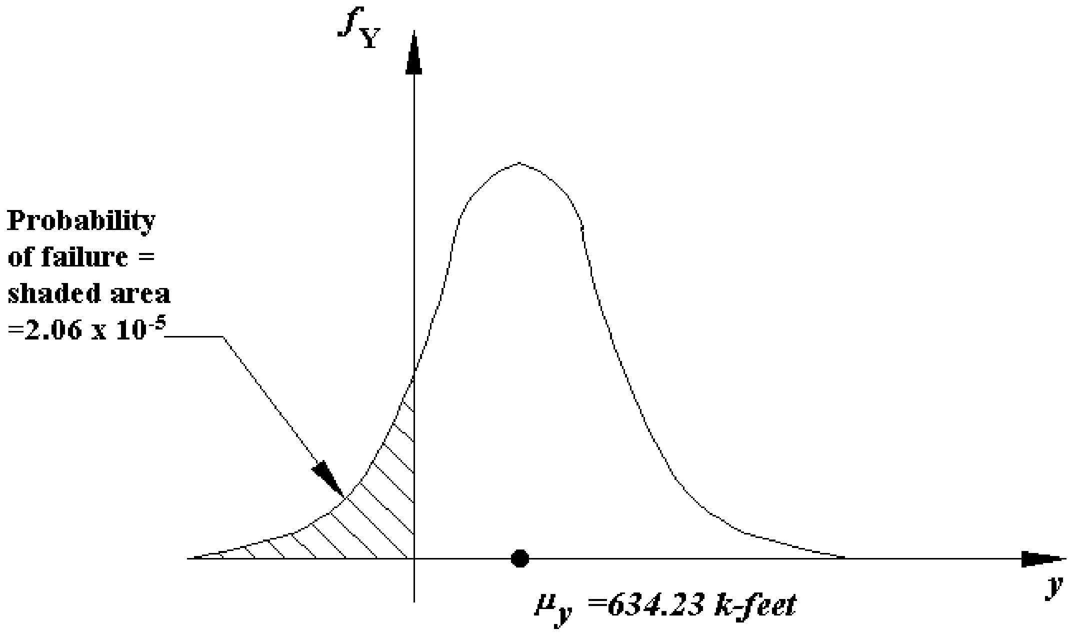
$$= 23892.5(k - feet)$$

$$\sigma_Y = 154.57(k - feet)$$

Thus, the probability of failure of girder is

$$P(Y < 0) = \Phi\left(\frac{0 - \mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-634.23}{154.57}\right) = \Phi(-4.1)$$

From Table 1.1.4.1, $= \Phi(-4.1) = 2.06 * 10^{-5}$.


f_Y
Probability
of failure =
shaded area
=2.06 x 10^{-5}
μ_y =634.23 k-feet
y

3.2 Nonlinear Functions of Random Variables

If a random variable, Y , is a nonlinear function of random variables $X_1, X_2,...X_n,$ the mean and variance of Y can be calculated approximately by the following steps.

Step 1: Use Taylor series expansion of Y to linearize the nonlinear function, Y , at a set of "design point values"
Nonlinear function $Y = f(X_1, X_2...X_n)$
Linearized function $Y \approx f(x_1^*, x_2^*,...x_n^*)$

$$+\sum_{i=1}^{n}(X_i - x_i^*)\frac{\partial f}{\partial X_i}\bigg|_{at\ (x_1^*, x_2^*,...x_n^*)} \quad (1)$$

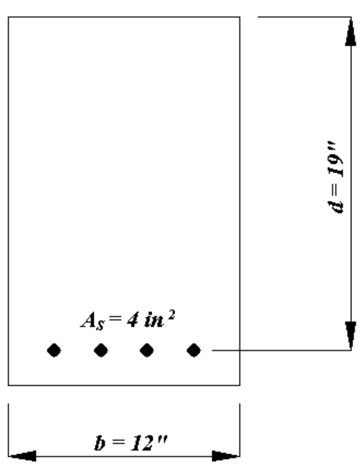
Step 2: If the nonlinearity of Y is not severe, the design points $(x_1^*, x_2^*,...x_n^*)$ may be approximately assumed to be the mean values of the random variables, i.e. $(\mu_{X_1}, \mu_{X_2},...\mu_{X_n})$.

Example 1.1.3.2.1: A 19"x12" beam with tension reinforcement area of $4\ in^2$ is subjected to a demand moment $Q.$ The moment capacity of the beam is

$$M_R = A_s f_y\left(d - \frac{a}{2}\right) = A_s f_y\left(d - 0.59\frac{A_s f_y}{f'cb}\right)$$

$$= A_s f_y d - 0.59\frac{(A_s f_y)^2}{f'cb}.$$

The random variables are $f_y, f'c,$ and $Q.$ Their statistical parameters are as follows:



$$\mu_{f_y} = 66\ ksi;\ V_{f_y} = 0.105;\ \therefore \sigma_{f_y} = \mu_{f_y} \cdot V_{f_y} = 6.93\ ksi$$

$$\mu_{f'c} = 4.16\ ksi;\ V_{f'c} = 0.14;\ \sigma_{f'c} = 0.5824\ ksi$$

$$\mu_Q = 3000\ k-in;\ V_Q = 0.12;\ \sigma_Q = 360\ k-in$$

Let $Y = M_R - Q$

$$= A_s f_y d - 0.59\frac{(A_s f_y)^2}{f'cb} - Q \qquad (1)$$

Equation (1) represents the limit state function and it is a nonlinear function. If $Y < 0$, failure will occur. Linearize equation (1) at the design points $(\mu_{f_y}, \mu_{f'c}, \mu_Q)$:

$$Y \approx 4.0\mu_{f_y} d - 0.59\frac{(4.0\mu_{f_y})^2}{\mu_{f'c} b} - \mu_Q$$

$$+ (f_y - \mu_{f_y})\left.\frac{\partial f}{\partial f_y}\right| \text{ evaluated at mean value}$$

$$+ (f'c - \mu_{f'c})\left.\frac{\partial f}{\partial f'c}\right| \text{ evaluated at mean value}$$

$$+ (Q - \mu_Q)\left.\frac{\partial f}{\partial Q}\right| \text{ evaluate at mean value}$$

$$\left.\frac{\partial f}{\partial f_y}\right|_{mean\ values} = \left. A_s d - 0.59\frac{2 f_y A_s^2}{f'cb}\right|_{mean\ values} = (4.0)(19) - 0.59\frac{(2)(66)(4.0)^2}{(4.16)(12)} = 51.04\ in^3$$

$$\left.\frac{\partial f}{\partial f'c}\right|_{mean\ values} = \left. 0.59\frac{(A_s f_y)^2}{(f'c)^2(b)}\right|_{mean\ values} = 0.59\frac{(4.0*66)^2}{(4.16)^2(12)} = 198.01\ in^3$$

$$\left.\frac{2f}{2Q}\right|_{mean\ values} = -1\Big|_{mean\ values} = -1$$

$$A_s \mu_{fy} d - 0.59 \frac{(A_s \mu_{fy})^2}{\mu_{f'c} b} - \mu_Q$$
$$= (4.0)(66)(19) - (0.59)\frac{(4.0*66)^2}{(4.16)(12)} - 3000 = 1192.27 \ \ k-in$$
$$\therefore Y \approx 1192.27 + (51.04)(f_y - 66)$$
$$+ (198.01)(f'c - 4.16) - (Q - 3000)$$
$$\approx 51.04 f_y + 198.01 f'c - Q$$

Therefore;

$$\mu_Y = \sum_{i=1}^{3} a_i \mu_{Xi} + 0$$
$$= (51.04)(66) + (198.01)(4.16) - (3000) + 0 = 1192.36 \ \ k-in$$
$$\sigma_Y^2 = \sum_{i=1}^{3} a_i^2 \sigma_{Xi}{}^2 = (51.04)^2 (6.93)^2$$
$$+ (198.01)^2 (0.5824)^2 + (360)^2 = 268007.72 \ \ (k-in)^2$$
$$\therefore \sigma_Y = 517.69 \ \ (k-in)$$

Thus, from Table 1.1.4.1 the probability of failure of beam is

$$P(Y < 0) = \Phi\left(\frac{0 - \mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-1192.36}{517.69}\right)$$
$$= \Phi(-2.303) = 1.07 * 10^{-2}$$

1.1.4 Structural Reliability Analysis

4.1 Probability of Failure

The basic structural reliability problem considers only one load effect Q resisted by the resistance R. Both Q and R are described by a known probability density function, $f_Q(\ )$ and $f_R(\ )$, respectively. A structural element will be considered to have failed if the resistance R is less than the load resultant Q acting on it. The probability P_f of failure of a structural element can be stated in the following ways:

$$P_f = P(R \le Q) \quad (1a)$$
$$= P(R - Q \le 0) \quad (1b)$$
$$= P(\frac{R}{Q} \le 1) \quad (1c)$$

or in general form

$$= P[G(R,Q) \le 0] \quad (1d)$$

Where $G(\ )$ is termed the limit state function and the probability of failure is identical to the probability of limit state violation. $G(R,Q)$ may be linear or nonlinear. The probability density functions f_R, f_Q for R and Q respectively are shown in Figure 1.1.4.1. In figure 1.1.4.2, equation (1) is represented by the hatched failure domain D, so that the failure probability becomes:

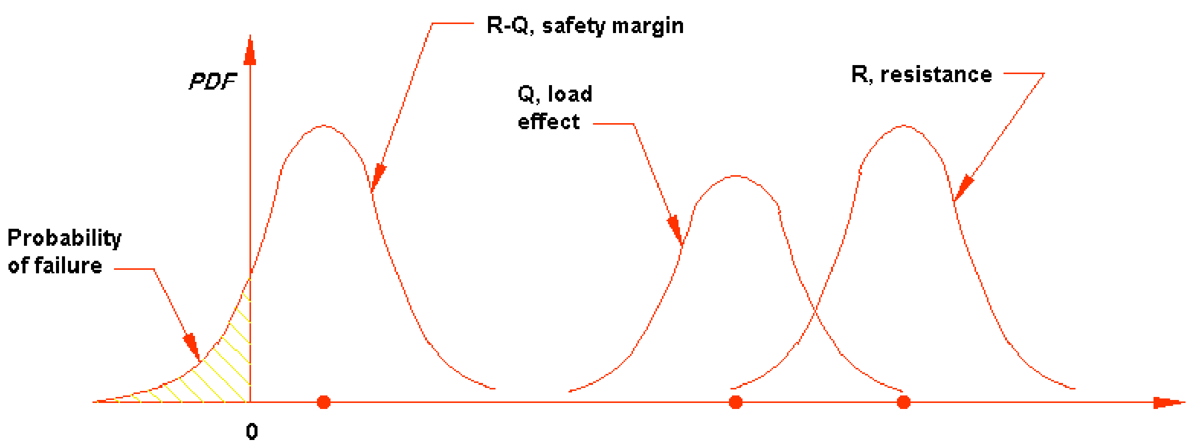


Figure 1.1.4.1 – PDF of load, resistance, and safety margin

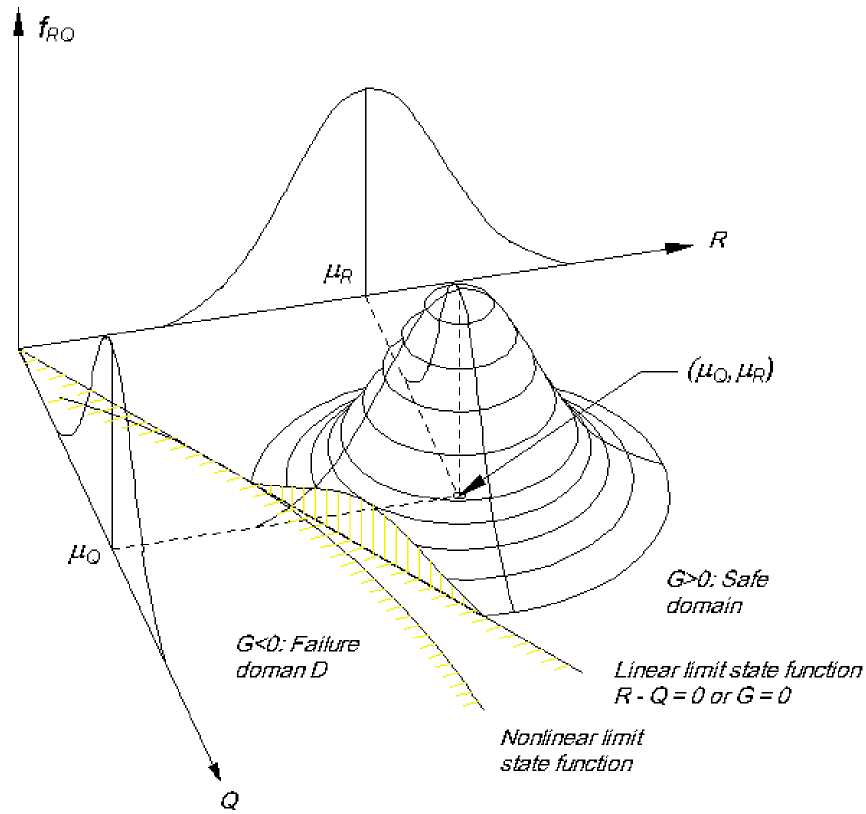


Figure 1.1.4.2 - Region D of integration for failure probability determination

$$P_f = P(R - Q \le 0) = \int_D \int f_{RQ}(r,q)dr \ dq \qquad (2)$$

when R and Q are independent, $f_{RQ}(r,q) = f_R(r) f_Q(q)$ so that equation (2) becomes:

$$P_f = P(R - Q \le 0) = \int_{-\infty}^{\infty} \int_{\infty}^{Q \ge r} f_R(r) f_Q(q)dr \ dq \qquad (3)$$

$$= \int_{-\infty}^{\infty} F_R(r) f_Q(r)dr \qquad (4)$$

where $F_R(r)$ is the cumulative probability of $R \le r$, or the probability that the actual resistance R of the member is less than some limit state value r. Current AASHTO LRFD design considers 4 different limit states. They are 1) Service limit state, 2) Fatigue and Fracture limit state, 3) Strength limit state and 4) Extreme event limit state.

From equations (2), (3), or (4), the probability of failure is calculated by integration of the joint density function over the failure domain (i.e. $G<0$). In general, it is very difficult to evaluate these integrals, especially when $G=0$ is nonlinear. Therefore, in practice, the probability of failure is calculated indirectly using an other procedure called “Safety Index” to quantify structural reliability. The safety index is described in the following section.

4.2 Safety (Reliability) Index

Figure 1.1.4.3 shows the top view of the Figure 1.1.4.2. The contours of joint probability function f_{RQ} may not be symmetric because the unit of measurement (such as standard deviation) of each random variable may be different (i.e. $\sigma_R \neq \sigma_Q$). However if R and Q are transformed into two standard non dimensional forms (i.e. standard normal distribution with zero mean and unit variance) as

$$Z_R = \frac{R - \mu_R}{\sigma_R}$$

$$Z_Q = \frac{Q - \mu_Q}{\sigma_Q}$$

the Figure 1.1.4.3 transforms to Figure 1.1.4.4. Z_R and Z_Q are called "reduced variables".

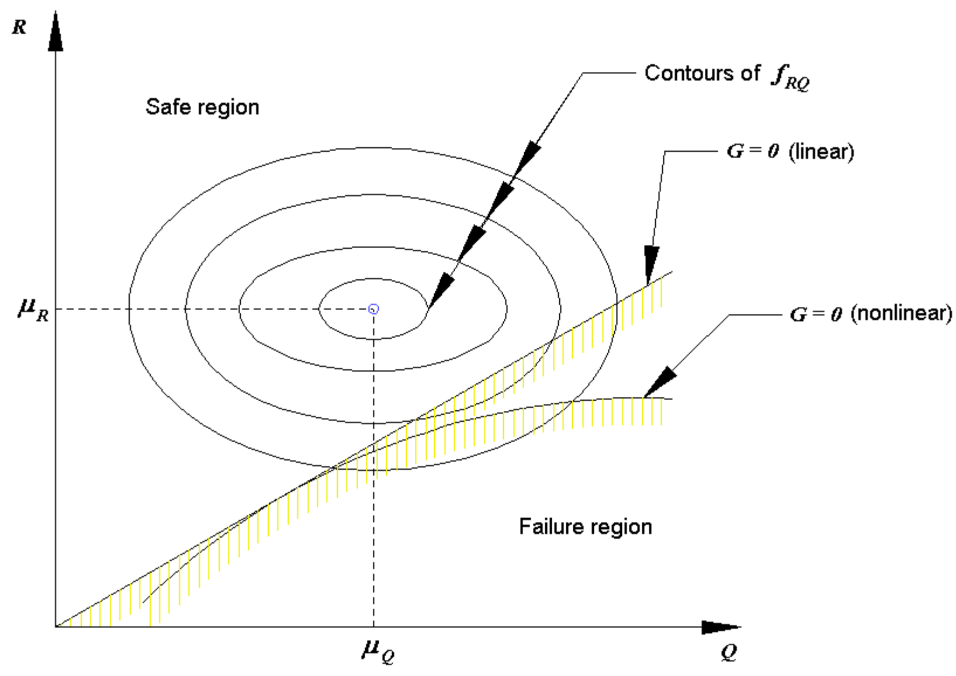


Figure 1.1.4.3 Limit State Surface in the space of R and Q.

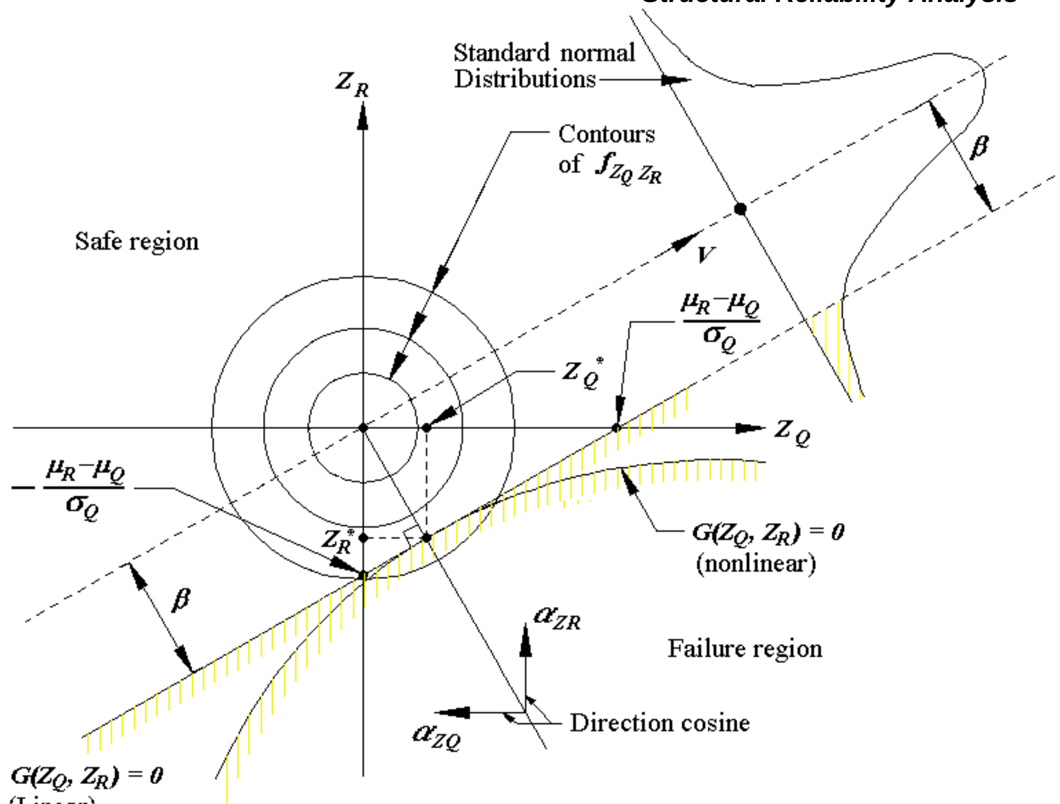


Figure 1.1.4.4 Limit State Surface in the space of standardized Z_Q and Z_R

The joint probability function $f_{Z_Q Z_R}$ is now a bivariate normal distribution and symmetrical about the origin. From Figure 1.1.4.4, the probability of failure, P_f, is equal to the integration of $f_{Z_Q Z_R}$ over the failure region along the V direction. However, by well-known properties of the bivariate normal distribution, the integration of $f_{Z_Q Z_R}$ along the V direction leads to a marginal standard normal distribution (see Figure 1.1.4.4).

Figure 1.1.4.4. indicates that

$$P_f = (P_f)_{\text{marginal standard normal distribution}} = \Phi(-\beta) \qquad (1)$$

Equation (1) shows that the P_f of equation (3) in Section 1.1.4.1 can be directly calculated by using standard normal distribution through reliability index β (see Figure 1.1.4.4). From Figure 1.1.4.4, it can be seen that the reliability index, β, is the shortest distance from the origin of Z_R and Z_Q coordinates. Thus

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \qquad (2)$$

If R and Q are normally distributed random variables, then, the probability of failure is

$$P_f = P(R - Q \le 0) = P(G < 0) = \Phi\left(\frac{0 - \mu_G}{\sigma_G}\right)$$

$$= \Phi\left(\frac{0 - (\mu_R - \mu_Q)}{\sqrt{\sigma_R^2 + \sigma_Q^2}}\right) = \Phi(-\beta) \qquad (3)$$

or $\beta = \Phi^{-1}(P_f) = \dfrac{\mu_G}{\sigma_G}$ (4)

The relationships between β and P_f are shown in Table 1.1.4.1

Table 1.1.4.1 Probability of Failure vs. β .

Reliability Index β	Reliability $S(=1-P_f)$	Probability of failure $P_f = \Phi(-\beta)$
0.0	0.500	$0.500 \times 10^{+0}$
0.5	0.691	$0.309 \times 10^{+0}$
1.0	0.841	$0.159 \times 10^{+0}$
1.5	0.9332	0.668×10^{-1}
2.0	0.9772	0.228×10^{-1}
2.2	0.9861	0.139×10^{-1}
2.3	0.98928	0.1072×10^{-1}
2.5	0.99379	0.621×10^{-2}
3.0	0.99865	0.135×10^{-2}
3.5	0.999767	0.233×10^{-3}
4.0	0.9999683	0.317×10^{-4}
4.1	0.99997938	0.2062×10^{-4}
4.2	0.99998668	0.1332×10^{-4}
4.5	0.99999660	0.340×10^{-5}
5.0	0.999999713	0.287×10^{-6}
5.5	0.9999999810	0.190×10^{-7}
6.0	0.999999999013	0.987×10^{-9}
6.5	0.9999999999598	0.402×10^{-10}
7.0	0.99999999999872	0.128×10^{-11}
7.5	0.999999999999681	0.319×10^{-13}
8.0	0.9999999999999389	0.611×10^{-15}

Equation (2) only shows the reliability index of two uncorrelated random variables R and Q.

Consider a linear limit state function of the form

$$G(X_1, X_2, ..., X_n) = a_0 + a_1 X_1 + a_2 X_2 + ... a_n X_n = a_0 + \sum_{i=1}^{n} a_i X_i \quad (5)$$

Where $a_i (i = 0,1,...,n)$ are constants and X_i are uncorrelated random variables. The reliability index β can be expressed as

$$\beta = \frac{a_0 + \sum_{i=1}^{n} a_i \mu_{Xi}}{\sqrt{\sum_{i=1}^{n} (a_i \sigma_{Xi})^2}} \quad (6)$$

Equation (6) shows that the reliability index depends only on the means and standard deviations of the random variables. There, β is also called a second-moment reliability index. Considering a nonlinear limit state function. $G(X_1, X_2, ... X_n)$, it can be linearized approximate by the Taylor series expansion at a design point $(X_1^*, X_2^*, ... X_n^*)$ along the $G = 0$ curve.

$$G(X_1, X_2, ... X_n) \approx G(X_1^*, X_2^*, ... X_n^*) \quad (7)$$

$$+ \sum_{i=1}^{n} (X_i - X_i^*) \frac{\partial g}{\partial X_i} \Bigg|_{evaluated\ at\ (X_1^*, X_2^*, ... X_n^*)}$$

Once $g(X_1, X_2, ... X_n)$ is linearized by eq. (7), similar to eq.(5), the reliability index β can be estimated as

$$\beta = \frac{g(X_1^*, X_2^*, ... X_n^*)}{\sqrt{\sum_{i=1}^{n} (a_i \sigma_{Xi})^2}} \text{ where } a_i = \frac{2G}{2X_i} \Bigg|_{\substack{evaluated\ at \\ design\ point \\ (X_1^*, X_2^*, ... X_n^*)}}$$

This can be illustrated by using two random variables R and Q as shown in Figure 1.1.4.5.

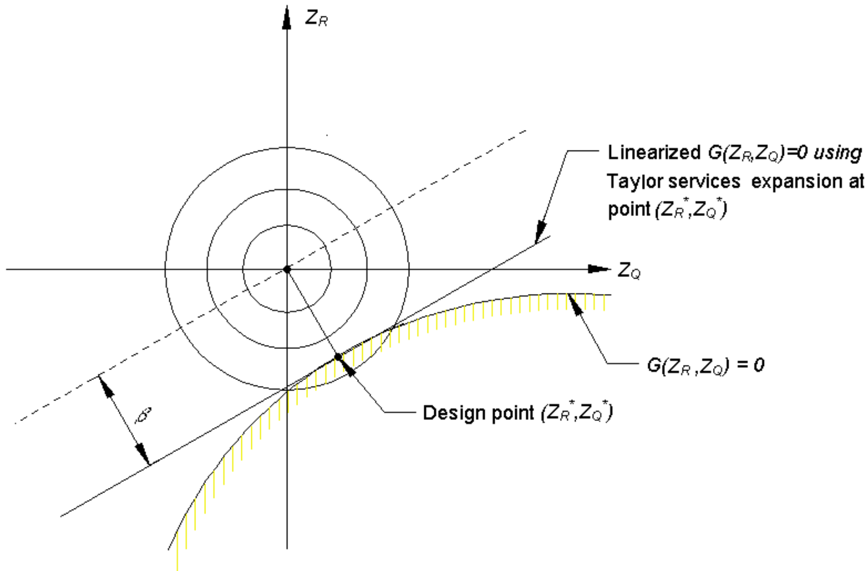


Figure 1.1.4.5 Design Point on the Failure Boundary

The design point is a point on the limit state function $G = 0$ from which the shortest distance, β , to the origin of the reduced variable space occurs. Since this design point is generally not known at this time, an integration technique called "Hasofer-Lind reliability index" must be used in order to find the location of the design point. The Hasofer-Lind reliability index integration technique can be briefly described as follows:

For n random variables, solve $(2n+1)$ simultaneous equations with $(2n+1)$ unknowns. The unknowns are

$\beta, \alpha_1, \alpha_2, \ldots \alpha_n, z_1^*, z_2^*, \ldots z_n^*$ where

$$\alpha_i = \frac{-\left.\frac{\partial g}{\partial z_i}\right|_{evaluated\ at\ design\ point}}{\sqrt{\sum_{j=1}^{n}\left(\left.\frac{\partial g}{\partial z_j}\right|_{evaluated\ at\ design\ point}\right)^2}} \tag{8}$$

$$\frac{\partial g}{\partial Z_i} = \frac{\partial g}{\partial X_i}\cdot\frac{\partial X_i}{\partial Z_i} = \frac{\partial g}{\partial X_i}\sigma_{X_i} \tag{9}$$

$$\sum_{i=1}^{n}(\alpha_i)^2 = 1 \tag{10}$$

$$z_i^* = \beta\alpha_i \tag{11}$$

$$g(z_1^*, z_2^*, \ldots z_n^*) = 0 \tag{12}$$

Equation (12) indicates that the design point $\left(z_1^*, z_2^*, \ldots z_n^*\right)$ is on the failure boundary $G = 0$.

Example 1.1.4.2.1: Find the reliability index of example 1.1.3.2.1 by Hasofer-Lind method.

Solution:

Define the limit state function:

From example 1.1.3.2.1, the limit state function is:

$$G = M_R - Q = 0$$

$$= A_s f_y d - 0.57 \frac{\left(A_s f_y\right)^2}{f'cb} - Q = 76 f_y - 0.7867 \frac{\left(f_y\right)^2}{f'c} - Q = 0$$

$$\therefore G = 76 f_y f'c - 0.7867\left(f_y\right)^2 - Qf'c = 0 \qquad (1)$$

The means and standard deviation of random variables are:

$$\mu_{fy} = 66 \ \ ksi; \ \ \sigma_{fy} = 6.93 \ \ ksi$$

$$\mu_{f'c} = 4.16 \ \ ksi; \ \ \sigma_{f'c} = 0.5824 \ \ ksi$$

$$\mu_Q = 3000 \ \ k - in; \ \ \sigma_Q = 360 \ \ k - in$$

Define the reduced variables

$$\left.\begin{aligned} Z_1 &= \frac{f_y - \mu_{fy}}{\sigma_{fy}} \Rightarrow f_y = \mu_{fy} + Z_1\sigma_{fy} = 66 + 6.93Z_1 \\ Z_2 &= \frac{f'c - \mu_{f'c}}{\sigma_{f'c}} \Rightarrow f'c = \mu_{f'c} + Z_2\sigma_{f'c} = 4.16 + 0.5824Z_2 \\ Z_3 &= \frac{Q - \mu_Q}{\sigma_Q} \Rightarrow Q = \mu_Q + Z_3\sigma_Q = 3000 + 360Z_3 \end{aligned}\right\} \qquad (2)$$

Substitute eq. (2) into eq. (1):

$$76\left[66 + 6.93Z_1\right]\left[4.16 + 0.5824Z_2\right] - 0.7867\left[66 + 6.93Z_1\right]^2 - \left[3000 + 360Z_3\right]\left[4.16 + 0.5824Z_2\right] = 0$$

$$\Rightarrow 1471.36Z_1 + 1174.12Z_2 - 1497.6Z_3 - 37.78Z_1^2 + 306.74Z_1Z_2 - 209.664Z_2Z_3 + 4959.7 = 0$$

3) Formulate g in terms of β and α_i:

$$Z_1^* = \beta\alpha_i$$

$$g = 1471.36\beta\alpha_1 + 1174.12\beta\alpha_2 - 1497.6\beta\alpha_3 - 37.78\beta^2\alpha_1^2 + 306.74\beta^2\alpha_1\alpha_2 - 209.664\beta^2\alpha_2\alpha_3 + 4959.7 = 0$$

$$\therefore \left[1471.36\alpha_1 + 1174.12\alpha_2 - 1497.6\alpha_3\right]\beta - \left[37.78\alpha_1^2 - 306.74\alpha_1\alpha_2 + 209.664\alpha_2\alpha_3\right]\beta^2 + 4959.7 = 0$$

$$\beta = \frac{\left[37.78\alpha_1^2 - 306.74\alpha_1\alpha_2 + 209.664\alpha_2\alpha_3\right]\beta^2 - 4959.7}{1471.36\alpha_1 + 1174.12\alpha_2 - 1497.6\alpha_3} \qquad (3)$$

4) Calculate α_i values:

$$\alpha_1 = \frac{-\left[1471.36 - 75.56\beta\alpha_1 + 306.74\beta\alpha_2\right]}{\sqrt{\begin{array}{l}\left[1471.36 - 75.56\beta\alpha_1 + 306.74\beta\alpha_2\right]^2 + \\ + \left[1174.12 + 306.74\beta\alpha_1 - 209.664\beta\alpha_3\right]^2 \\ + \left[-1497.6 - 209.664\beta\alpha_2\right]^2\end{array}}} \qquad (4)$$

$$\alpha_2 = \frac{-\left[1174.12 + 306.74\beta\alpha_1 - 209.664\beta\alpha_3\right]}{\sqrt{\begin{array}{l}\left[1471.36 - 75.56\beta\alpha_1 + 306.74\beta\alpha_2\right]^2 + \\ + \left[1174.12 + 306.74\beta\alpha_1 - 209.664\beta\alpha_3\right]^2 \\ + \left[-1497.6 - 209.664\beta\alpha_2\right]^2\end{array}}} \qquad (5)$$

$$\alpha_3 = \frac{-\left[-1497.6 - 209.664\beta\alpha_2\right]}{\sqrt{\begin{array}{l}\left[1471.36 - 75.56\beta\alpha_1 + 306.74\beta\alpha_2\right]^2 + \\ + \left[1174.12 + 306.74\beta\alpha_1 - 209.664\beta\alpha_3\right]^2 \\ + \left[-1497.6 - 209.664\beta\alpha_2\right]^2\end{array}}} \qquad (6)$$

5) Start iteration with a guess for $\beta, \alpha_1, \alpha_2,$ *and* α_3 :

Let $\alpha_1 = \alpha_2 = -\sqrt{0.333} = -0.58;\ \ \alpha_3 = \sqrt{0.333} = 0.58$

and let $\beta = 3$

The iterations are summarized in Table 1.1.4.2. The reliability index β changes very little between iterations 4 and 5, so the solution has converged. The final $\beta = 2.285$. By comparing with the approximated reliability index $\beta = 2.303$ in example 1.1.3.2.1, the approximated β of 2.303 is close to the actual β of 2.285.

Table 1.1.4.2 Integrations for Example 1.5.2.1

Iteration	β	α_1	α_2	α_3
1	2.667	-0.6759	-0.1742	0.7161
2	2.324	-0.7187	-0.1083	0.6869
3	2.290	-0.7162	-0.1540	0.6806
4	2.285	-0.7124	-0.1650	0.6821
5	2.285	-0.7115	-0.1674	0.6825

The Hasofer-Lind reliability index iteration technique described here is mainly for the normal random variables with nonlinear limit state function. If some of the random variables are not normal random variables, a procedure call “Rackwitz-Fiessler” procedure can be used to calculate the “equivalent normal” values of the mean and standard deviation for each non normal random variable. Once the equivalent normal parameters have been calculated, the basic steps in the iteration procedure are the same as those in the Hasofer-Lind iteration technique. The details of Rackwitz-Fiessler procedure are not described in here.

1.1.5 Monte Carlo Simulation

In Section 1.1.4, the probability of safety (reliability) of a structural member is evaluated by the reliability index. It is very difficult to perform reliability analysis of a member with consideration of complicated nonlinear (inelastic) behavior by using reliability index technique. The probabilistic problems involving complicated nonlinear behavior of a member can be solved by Monte Carlo Simulation.

The Monte Carlo Simulation is a technique to artificially simulate a large number of data without doing any physical testing, and then observe the number of times some event of interest occurs. To generate the artificial sample data, certain probability distribution of the important parameters need to be known first.

5.1 Generation of Normal Random Numbers

Since the normal probability distribution plays an important role in the structural reliability analysis, it is important to generate normal random numbers by Monte Carlo Simulation. The steps of simulating normal random numbers are shown as follows.

Step 1 – Generation of Uniform Distributed Random Number:

Most mathematical computer programs have random number generators for generating uniform distributed random number as described in Section 1.1.2.3.

Table 1.1.5.1 gives a short list of random numbers with values between 0 and 1.

Table 1.1.5.1 – Generated Uniform Distributed Random Numbers

0.050203	0.269082	0.442000
0.619129	0.472640	0.833705
0.872402	0.422864	0.412275
0.376568	0.467299	0.145451
0.139927	0.415784	0.849178
0.318491	0.523667	0.598193
0.987671	0.243629	0.561388
0.033265	0.741569	0.169408
0.234626	0.408673	0.967040
0.623157	0.021790	0.864834
0.957884	0.547472	0.332286
0.518906	0.749779	0.849239
0.442305	0.681600	0.834803
0.445845	0.140538	0.885769
0.834284	0.888607	0.359783
0.811213	0.105136	0.607227
0.935728	0.067202	0.705435
0.450423	0.864772	0.841975
0.579058	0.363628	0.784091
0.662648	0.863948	0.491226
0.039918	0.858242	0.557054
0.414075	0.432234	0.934263
0.103214	0.412091	0.087985
0.112308	0.749199	0.242988
0.821833	0.260933	0.012574

Step 2 – Generation of Standard Normal Random Numbers:

To generate a set of standard normal random numbers $z_1, z_2, ...z_n$, a corresponding set of uniform distributed random numbers $u_1, u_2, ...u_n$ between 0 and 1 need to be generated first. For each u_i, the corresponding z_i is

$$z_i = \Phi^{-1}(u_i) \tag{1}$$

where Φ^{-1} is the inverse of the standard normal cumulative distribution function. Equation (1) can be graphically explained by Figure 1.1.5.1.

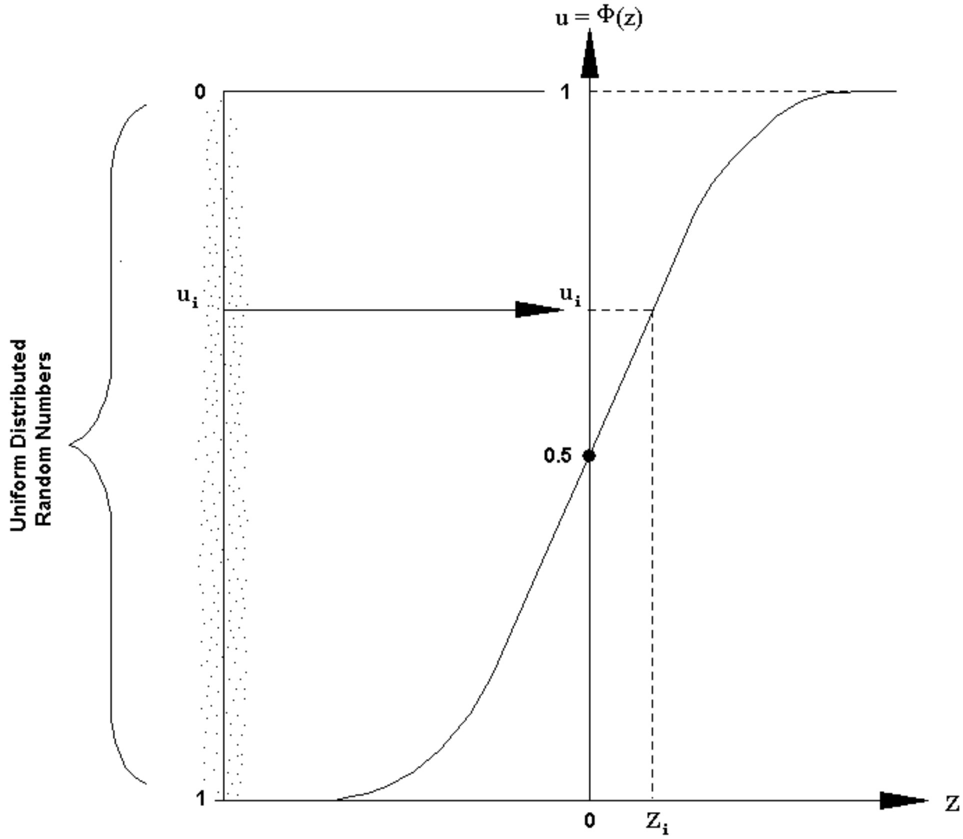


Figure 1.1.5.1 Generation of Standard Normal Random Numbers

Step 3 – Generation of Normal Random Numbers:

Let a normally distributed random variable X with mean μ_X and standard deviation σ_X, then from section 1.1.2.2,

$$X = \mu_X + Z\sigma_X \tag{2}$$

or

$$x_i = \mu_X + z_i\sigma_X \tag{3}$$

Therefore a random value x_i can be obtained by using a generated standard random number z_i described in step 2.

Example 1.1.5.1: Using Monte Carlo Simulation technique, find the probability of failure of a girder shown in example 1.1.3.1.1. The girder is subjected to a concentrated dead load, P, and a dead load, DL. The moment capacity of the girder is M_R. The statistical parameter of DL, P, and M_R are

$$\mu_{DL} = 1.6(k/feet);\ \sigma_{DL} = 0.16(k/feet)$$
$$\mu_P = 0.3(kips);\ \sigma_P = 0.045(kips)$$
$$\mu_{M_R} = 1200(k-feet);\ \sigma_{M_R} = 144(k-feet)$$

Solution: From example 1.1.3.1.1 the demand moment at center of the girder is

$$M = \frac{(DL)(L)^2}{8} + \frac{PL}{4} = 351.12(DL) + 13.25(P)$$

the limit state function is

$$Y = M_R - M = M_R - 351.12(DL) - 13.25(P) \quad (1)$$

For simplicity, use only 5 values of the M_R, 5 values of DL and 5 values of the P using Monte Carlo Simulation. From table 1.1.5.1, the first 5 uniform random numbers will be used to calculate five values of M_R, the second 5 uniform random numbers will be used to calculate values of DL, and the third 5 uniform random numbers will be used to calculate five values of P. The simulated values of M_R, DL, and P are summarized in Table 1.1.5.2.

Table 1.1.5.2 Simulation for M_R, DL, and P

u_i	z_i	$M_{Ri} = 1200 + 144z_i$
0.0502	-1.6429	963.42
0.6191	0.3032	1243.66
0.8724	1.1378	1363.84
0.3765	-0.3145	1154.71
0.1300	-1.0806	1044.39
u_i	z_i	$DL_i = 1.6 + 0.16z_i$
0.31849	-0.4719	1.5245
0.9877	2.2467	1.9595
0.03326	-1.8348	1.3064
0.2346	-0.7237	1.4842
0.6231	0.3138	1.6502
u_i	z_i	$P_i = 0.3 + 0.045z_i$
0.9579	1.727	0.3777
0.5189	0.05	0.3023
0.4423	-0.145	0.2935
0.4458	-0.135	0.2939
0.8343	0.97	0.03436

From Table 1.1.5.2, 5 generated values of Y are

$$Y = M_R - 351.12(DL) - 13.25(P)$$

$$= \begin{cases} 423.133 \\ 551.635 \\ 901.25 \\ 629.68 \\ 464.51 \end{cases}$$

The sample mean and sample standard deviation can be obtained using equations (4) and (5) in Section 1.1.2.2.

$$\mu_Y = \frac{1}{n}\sum_{i=1}^{5} y_i = \frac{1}{5}(2970.198) = 594.04 \ (k - feet)$$

$$\sigma_Y = \sqrt{\frac{\left(\sum_{i=1}^{5} y_i^2\right) - 5(\mu_Y)^2}{5-1}} = \sqrt{\frac{1907860.71 - 1764417.61}{4}} = 189.3 \ (k - feet)$$

Thus, the probability of failure of the girder is

$$P(Y < 0) = \Phi\left(\frac{0 - \mu_Y}{\sigma_Y}\right) = \Phi(-3.14) = 8.45 \times 10^{-4}$$

Comparing with the theoretical $\mu_Y = 634.23 \ k - feet$ and $\sigma_Y = 154.57 \ (k - feet)$ in example 1.1.3.1.1, it can be seen that μ_Y and σ_Y by Monte Carlo Simulation are not close to the theoretical μ_Y and σ_Y respectively. It is because only 5 sample values are used in the example. The accuracy of the Monte Carlo Simulation increases as the number of sample values increases.

1.1.6 Statistical Load Models

Loads considered in bridge design are dead load, live load (static and dynamic), environmental loads (temperature, wind, earthquake) and other loads (collision, braking, etc.). The statistical load models are developed using the available statistical data and surveys. Loads are treated as random variables. Their variation is described by the cumulative distributions function (CDF), mean value and coefficient of variation. The relationship among various load parameters is described in terms of the coefficients of correlations.

It is assumed that the economic life span for a newly designed bridge is 75 years. Therefore extreme values of live load and environmental loads are extrapolated accordingly from the available data base described as follows:

6.1 Dead Load

Dead load, D, is the gravity load due to self weight of structural and nonstructural elements. Dead loads can be categorized as follow:

D_1 = weight of factory made elements (steel, precast concrete members),
D_2 = weight of cast-in-place concrete members,
D_3 = weight of the wearing surface (asphalt),
D_4 = miscellaneous weight (e.g. railing, luminaries).

$D_1 \sim D_4$ are modeled as normal random variables. The statistical parameters developed are listed in Table 1.1.6.1. In the table, bias factor is defined as the ratio of mean value to the nominal value. As mentioned previously, the CDF of each random variable $(D_1, D_2, D_3\ or\ D_4)$ is generated by statistical data and/or surveys. The CDF can be expressed in terms of standard normal probability paper. Figure 1.1.6.1 shows the probability paper of D_3. It shows that the mean value (μ) is about 1 and the mean plus a standard deviation $(\mu + \sigma)$ is 1.25, which leads to the coefficient of variation $V = \frac{\sigma}{\mu} = \frac{0.25}{1} = 0.25$ as shown in Table 1.1.6.1

Table 1.1.6.1 Statistical Parameters of Dead Load

Component	Bias Factor	Coefficient of Variation
Factory-made members	1.03	0.08
Cast-in-Place members	1.05	0.10
Asphalt	3.5 inch*	0.25
Miscellaneous	1.03-1.05	0.08-0.10

*mean thickness

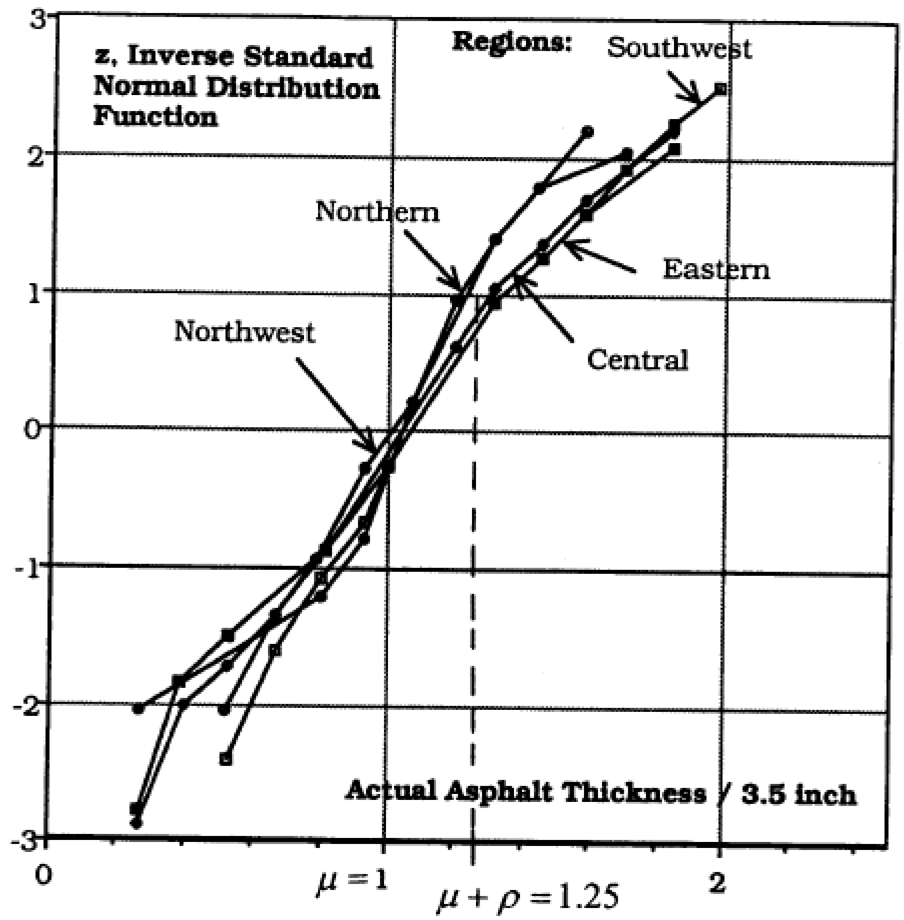


Figure 1.1.6.1 Cumulative Distribution Functions of Asphalt Thickness

6.2 Live Load

The statistical live load model is mainly based on the available truck survey data. About 10,000 selected trucks being heavily loaded were measured. Even though 10,000 trucks are considered, it is a very small amount compared to the actual number of heavy vehicles in a 75 year bridge life time. Therefore, the maximum load effects corresponding to a 75 year life are calculated by extrapolation of available truck survey data. For each truck in the survey, bending moments, M , and shear forces, V , are calculated for a wide range of spans. Simple spans and two equal continuous spans are considered. The moments and shears are calculated in terms of the standard HS20 truck or lane loading, whichever governs. Typical continuous spans are plotted on normal probability paper in Figure 1.1.6.2. The maximum moments for various time periods are extrapolated from figure 1.1.6.2 and shown in Figure 1.1.6.3. Similar probability paper for the maximum shears for various time periods can also be generated.

Let N be the total number of trucks in time period, T . Assume the surveyed trucks represent about two weeks of traffic. Therefore, if $T = 75$ years, the number of trucks, N , will be about 2,000 times larger than in the survey. This will result in $N = 20$ million trucks. The probability level corresponding to N is $1/N$, and for $N = 20$ million, the probability is $1/20{,}000{,}000 = 5\times10^{-8}$, which corresponds to $Z = 5.33$ on the vertical scale, as shown in Figure 1.1.6.3.

The mean maximum moments corresponding to various periods of time can be read from the graph. For example, for a 120' span and $T = 75$ years, the mean maximum moment is 1.2*(HS20 moment). The number of trucks passing through the bridge in 5 years is about 1,500,000. This corresponds to $z = 4.83$ on the vertical scale, and the resulting moment is 1.15* (HS20 moment). Similar calculations can be performed for other periods of time. The difference between the mean maximum 50 year moment and the mean maximum 75 year moment is about 1%.

To predict the mean and standard deviation for truck moments (or shears) in the time period of 75 years, first find the average probability of truck moments (or shears), P_f , which exceed the nominal moment (or shear) capacities of the bridges in the period of 2 weeks of surveying time (For example, $P_f = 10^{-4.86}$). Then the predicted probability of truck moments (or shears) which exceed the nominal moment (or shear) capacities is assumed to be:

$$P_f = \frac{75\ \ years \times 12\ \ months/year \times 4\, weeks/months}{2\ \ weeks}$$

$$\times 10^{-4.86} = 1800 \times 10^{-4.86} = 0.025$$

The *PDF* of *75 years* can then be estimated by shifting the original *PDF* based on two weeks to the right (see Figure 1.1.6.3B) to meet $P_f = 0.025$ and assuming the mean value of the *PDF* in *75 years* is equal to the mean maximum moment (or shear) in *75 years* from the probability paper in Figure 1.1.6.3. Thus the coefficient of variation for the maximum truck moments (or shears) in *75 years* can be obtained by converting the *PDF* of *75 years* to the corresponding probability paper.

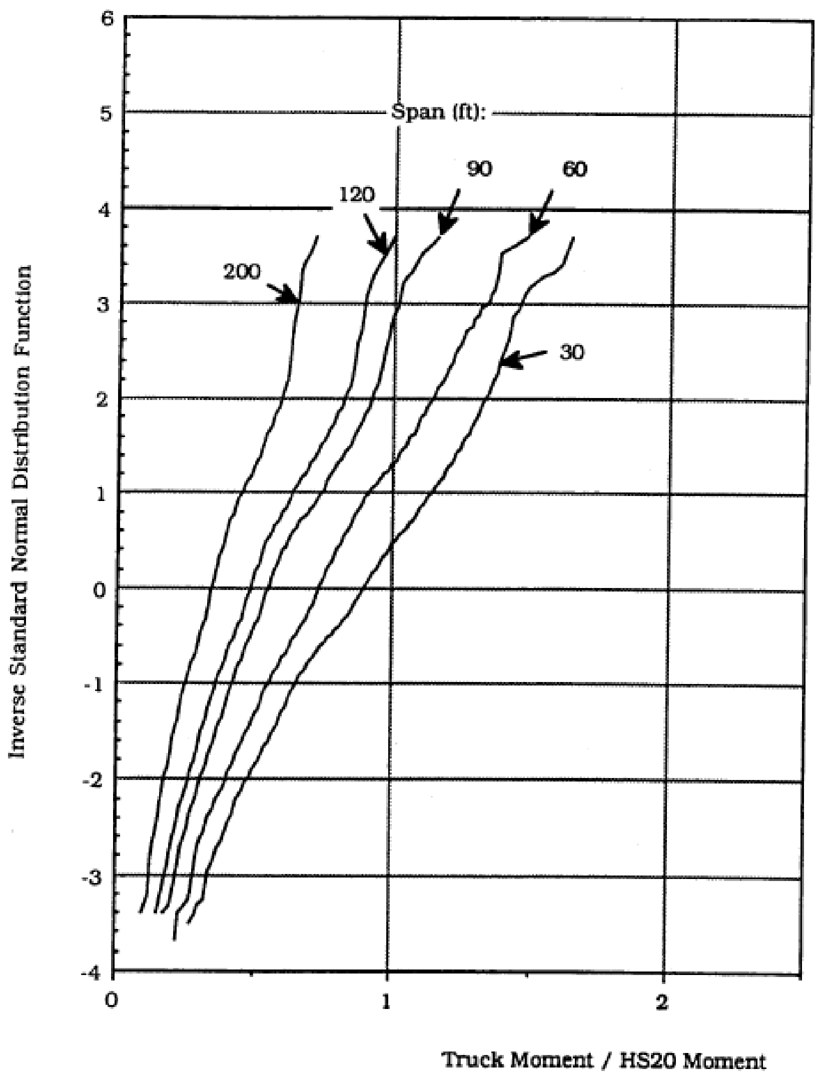


Figure 1.1.6.2 Negative Moments from Truck Survey for Two Equal Continuous Spans.

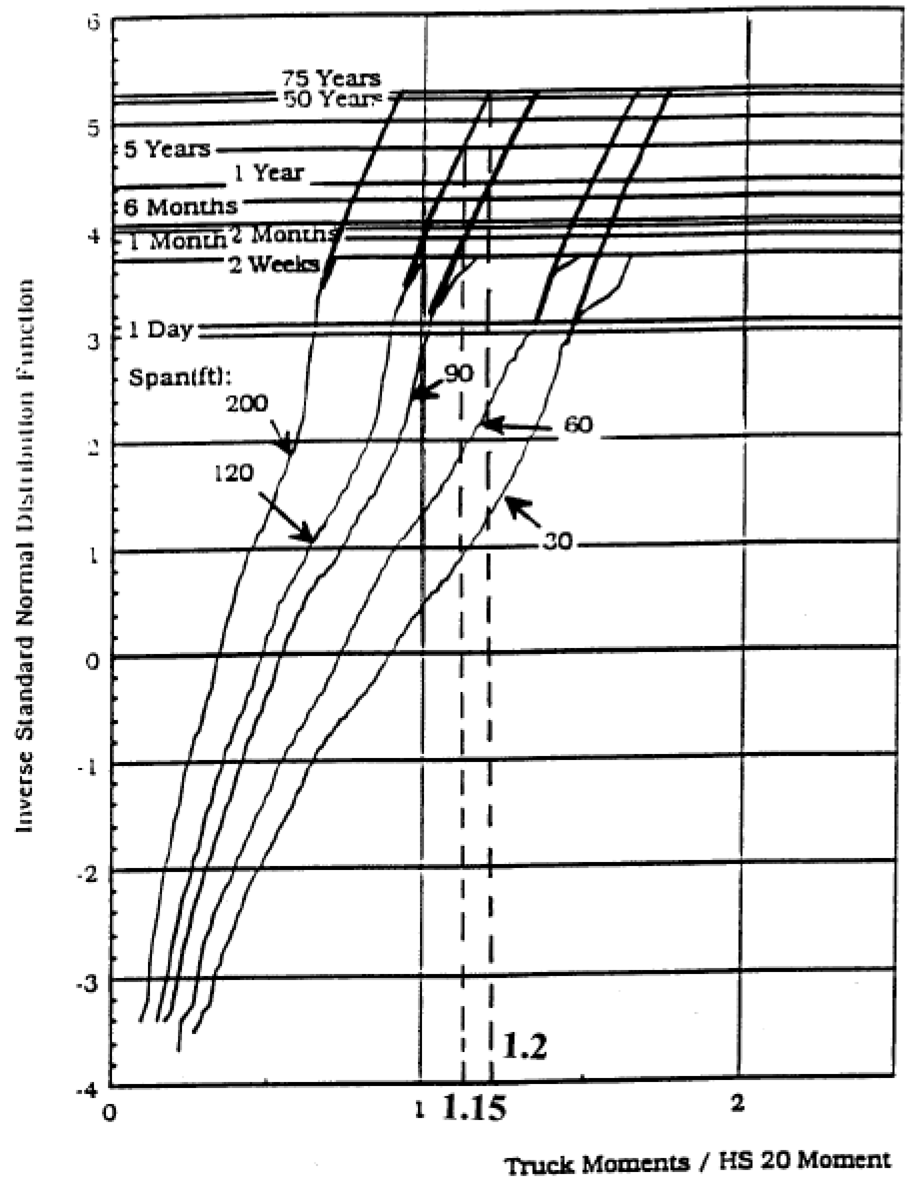


Figure 1.1.6.3 Extrapolated Negative Moments for Two Equal Continuous Spans.

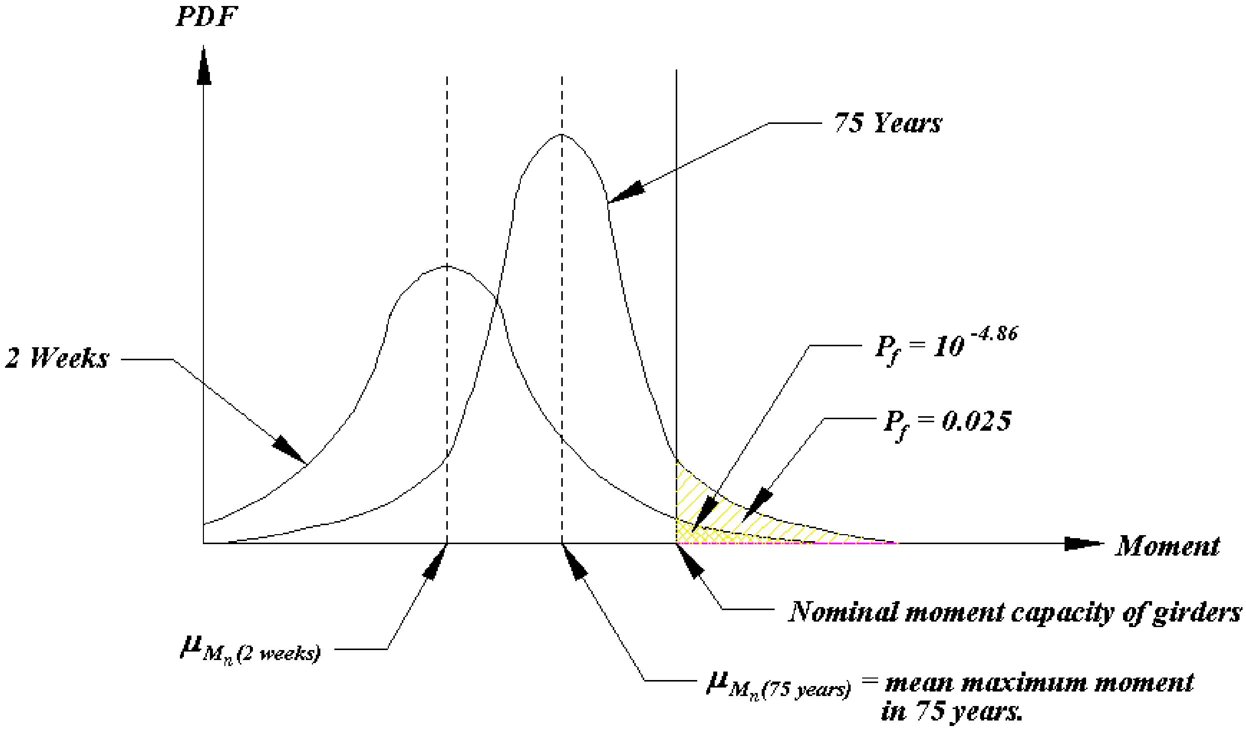


Figure 1.1.6.3B *PDF* of the maximum truck moments in 75 years.

6.3 LRFD HL-93 Design Live Load

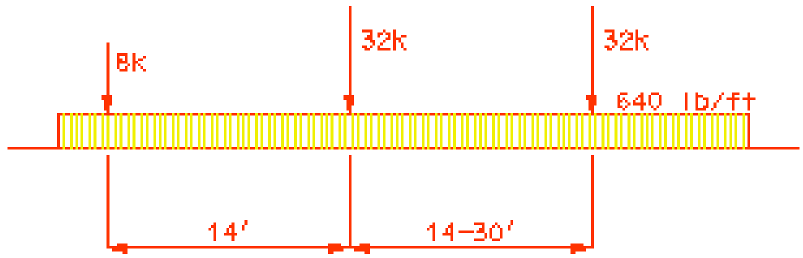
The objective in the selection of the live load model for the LRFD bridge design code is a uniform ratio of the nominal (design) moments (or shears) and the mean maximum 75 year moments (or shears) as described in Section 1.1.6.2. Various live load models were considered. For the considered models, the ratios of moments and shears were calculated for a wide range of spans. Good results were obtained for a model (called HL-93 design live load) which combines the HS20 truck with a uniformly distributed load of 640 lb/ft. For shorter spans, a tandem of two equal axles, each 25 kips, spaced at 4 ft, also combined with a uniform load of 640 lb/ft, is specified. For negative moment in continuous spans, the HL-93 design live load (per lane) is the larger of:

(a) One HS20 truck plus a uniformly distributed loading of 640 lb/ft,

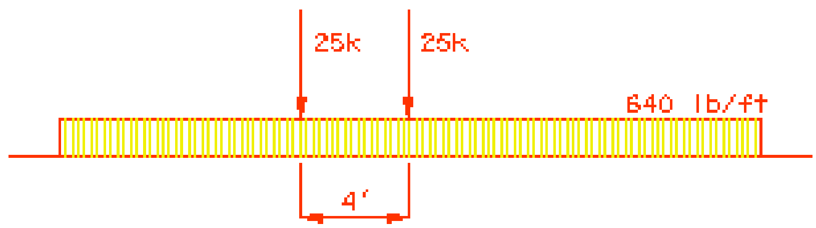
(b) 90% of the effect of two HS20 truck, placed in two different spans, with headway distance of at least 50 ft, plus 90% of the uniformly distributed loading of 640 lb/ft. The headway distance, 50 ft, corresponds to the minimum value for moving vehicles.

The HL-93 design live load is shown in Figure 1.1.6.4. The mean-to-nominal ratio (bias factor) of live load is equal to the ratio of the mean maximum 75 year load effect (described in Section 1.1.6.2) and the design value. The calculated bias factors for HS20 loading and the HL-93 loading are shown in Figure 1.1.6.5 for negative moment in Continuous Spans. It can be seen that the bias factor varies as a function of span, however, the variation is reduced for the HL-93 live load.

(a) Truck and Uniform Load



(b) Tandem and Uniform Load



(c) Alternative Load for Negative Moment (reduce to 90%)

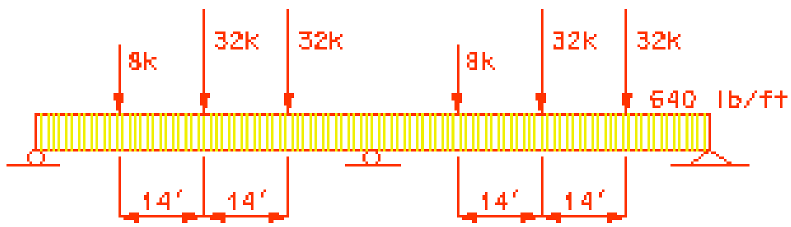


Figure 1.1.6.4 HL-93 Design Live Load in LRFD Code

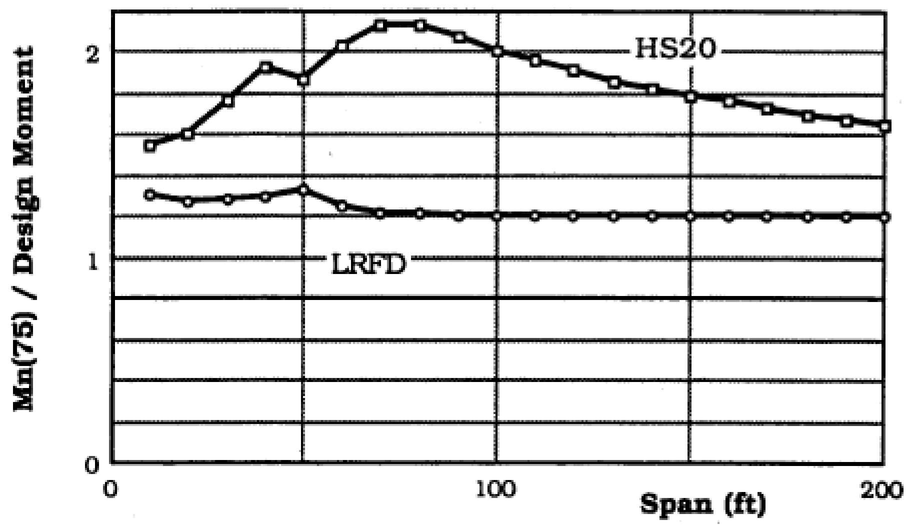


Figure 1.1.6.5 Bias Factors for Negative Moment; Ratio of Mn(75)/Mn(HL-93) and Mn(75)/Mn(HS20).

6.4 Load Combinations

For each load component, Q_i, if a factored load value, $\gamma_i Q_{ni}$, is defined, for example, as not being exceeded 2.5% of the load component in 75 years, the factored load $\gamma_i Q_{ni}$ can be shown in the following probability density function Q_i (Figure 1.1.6.6). Now the statistical parameters for individual probability density function, Q_i, such as dead load or live load can be obtained as described in Section 1.1.6.1 and 1.1.6.2, respectively.

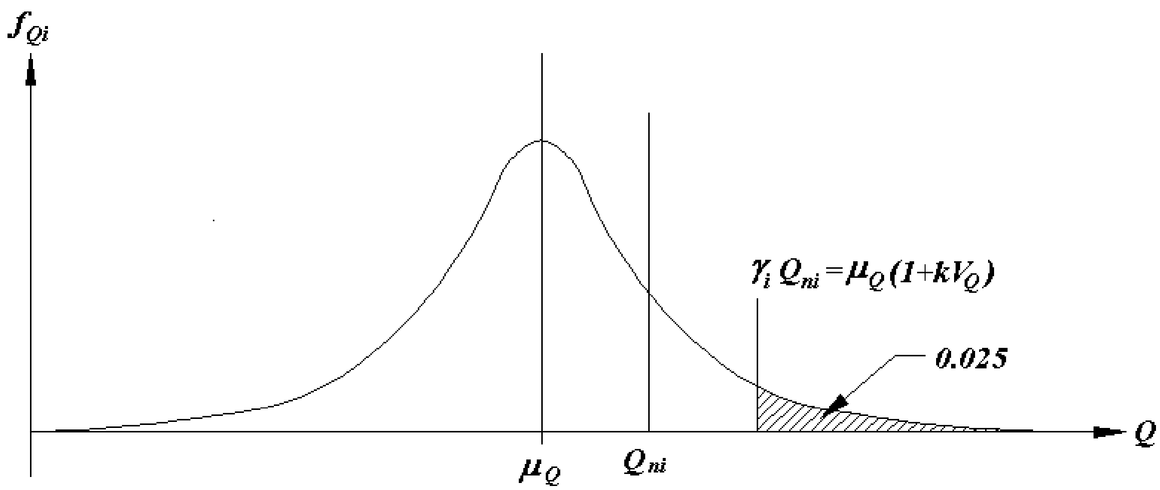


Figure 1.1.6.6 Probability density function, f_{Qi}, of load, Q_i; mean load, μ_Q, nominal (design) load, Q_{ni}, and factored load, $\gamma_i Q_{ni}$.

In the figure, the shaded area is equal to the probability of exceeding the factored load value.

From Figure 1.1.6.6,

$$0.025 = \Phi\left(\frac{\gamma_i Q_{ni} - \mu_Q}{\sigma_Q}\right)$$

$$\frac{\gamma_i Q_{ni} - \mu_Q}{\sigma_Q} = 2 = k \text{ (from Table 1.1.4.1)}$$

$$\gamma_i Q_{ni} - \mu_Q = k\sigma_Q$$

$$\gamma_i Q_{ni} = \mu_Q + k\sigma_Q$$

$$= \mu_Q\left(1 + k\frac{\sigma_Q}{\mu_Q}\right)$$

$$= \mu_Q\left(1 + kV_Q\right)$$

Therefore the load factor $\gamma_i = \frac{\mu_Q}{Q_{ni}}\left(1 + kV_Q\right)$

$= \lambda_i\left(1 + kV_Q\right)$ when λ_i = bias factor (mean load to nominal load ratio).

Note that the statistical parameters for individual probability density function, f_{Qi}, such as dead load or live load can be calculated through the processes described in Section 1.1.6.1 and 1.1.6.2, respectively. These parameters of load components are summarized in Table 1.1.6.2.

Various sets of load factors, corresponding to different values of k, are presented in Table 1.1.6.3. The relationship is also shown in Figure 1.1.6.7.

Recommended values of load factors correspond to k = 2. For simplicity of the designer, one factor is specified for D_1 and D_2, $\gamma = 1.25$. For D_3, weight of asphalt, $\gamma = 1.50$. For live load and impact, the value of load factor corresponding to k = 2 is $\gamma = 1.60$. Although a more conservative value of $\gamma = 1.70$ is proposed for the LRFD code, $\gamma = 1.75$ is chosen in the current LRFD code. In a similar matter, other load factors can be derived (such as wind load on structure).

Table 1.1.6.2 Parameters of Bridge Load Components.

Load Component	Bias Factor	Coefficient of Variation
Dead load, D_1	1.03	0.08
Dead load, D_2	1.05	0.10
Dead load, D_3	1.00	0.25
Live load (with impact)	1.10-1.20	0.18

Table 1.1.6.3 Considered Sets of Load Factors.

Load Component	k = 1.5	k = 2.0	k = 2.5
Dead load, D_1	1.15	1.20	1.24
Dead load, D_2	1.20	1.25	1.30
Dead load, D_3	1.375	1.50	1.65
Live load (with impact)	1.40-1.50	1.50-1.60	1.60-1.70

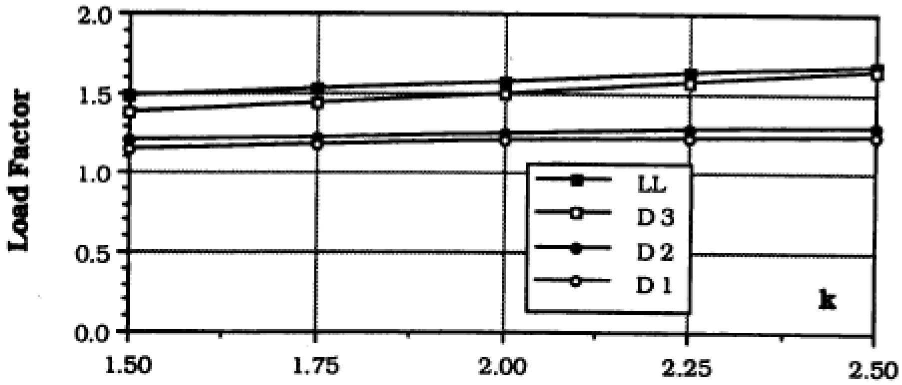


Figure 1.1.6.7 Load Factors vs. k .

The total load in a bridge is a joint effect of dead load D, live load plus impact $L+I$, environmental loads E (wind, snow, ice, earthquake, earth pressure, water pressure), and other loads A (collision forces, braking forces, etc.).

$$Q = D + (L+I) + E + A \qquad (1)$$

The distribution of the joint effects is based on the so called Turkstra's rule. Turkstra observed that a combination of several load components reaches its extreme when one of the components takes an extreme value and all other components are at their average (arbitrary-point-in-time) level. For example, the combination of live load with earthquake produces a maximum effect for the lifetime T , when either,

1. Earthquake takes its maximum expected value for T and live load takes its maximum expected value corresponding to the duration of earthquake (about 30 seconds), or
2. Live load takes its maximum expected value for T and earthquake takes its maximum expected value corresponding to duration of this maximum live load (time of truck passage on the bridge).

In practice, the expected value of an earthquake in any short time interval is almost zero. The expected value of truck load for a short time interval depends on the class of the road. For a very busy highway it is likely that there is some traffic at any point in time. Therefore, the maximum earthquake may occur simultaneously with an average truck passing through the bridge.

In the general case, Turkstra's rule can be expressed as follows,

$$Q(\max) = \max \ Q_i \text{ for } i = 1, 2, 3 \text{ and } 4 \qquad (2)$$

Where:

$$Q_1 = D(\max) + (L+I)(ave) + E(ave) + A(ave)$$
$$Q_2 = D(ave) + (L+I)(\max) + E(ave) + A(ave)$$
$$Q_3 = D(ave) + (L+I)(ave) + E(\max) + A(ave)$$
$$Q_4 = D(ave) + (L+I)(ave) + E(ave) + A(\max)$$

In all cases, the average load value is calculated for the period of time corresponding to the duration of the maximum load. The formula can be extended to include various components of D, E, and A. Since dead load does not vary with time, the maximum and average values are the same.

The probability of an earthquake EQ or heavy wind W, occurring in a short period of time is very small. Therefore, simultaneous occurrence of EQ and W is not considered. As a result, the number of load combinations considered in the code can be reduced as follows,

$$Q_{\max} = D + \max\begin{pmatrix} (L+I)\max \\ W_{\max;} \\ (L+I)_{4hour} + W_{daily} \\ EQ_{\max} \end{pmatrix} \qquad (3)$$

where $(L+I)_{\max} =$ maximum 75 year $(L+I)$; $(L+I)_{4hour} =$ maximum 4 hour $(L+I)$; $W_{\max} =$ maximum 75 year wind; $W_{daily} =$ maximum daily wind; $EQ_{\max} =$ maximum 75 year earthquake.

For example, the mean maximum 4 hour live load moment for span length of 200’, $(L+I)_{4hour}$, can be read directly from figure 1.1.6.2 for $z = 2.58$ (maximum of 200 trucks, so the probability level is $\frac{1}{200} = 0.005$, which corresponds to $z = 2.58$ (see Table 1.1.4.1)). Form figure 1.1.6.2, the maximum 4 hour live load moment is about (0.7)(HS20 moment). From Figure 1.1.6.3, the maximum 75 year live load moment is about (0.9)(HS20 moment).

Therefore, if the load factors for the first load combination are:

$$1.25D + 1.70(L+I) \qquad (4)$$

and for the second one they are,

$$1.25D + 1.40W \qquad (5)$$

then for the third combination, the load factors are,

$$1.25D + 1.35(L+I) + 0.45W \qquad (6)$$

where live load factor $= \left(\frac{0.7}{0.9}\right)(1.70) = (0.778)(1.70) = 1.32 \approx 1.35$ (mean maximum 4 hour truck is about 0.77 of the mean maximum 75 year truck); wind load factor $= (0.33)(1.4) = 0.46$ (mean maximum daily wind is 0.33 of the mean maximum 75 year wind). Based on eq. (6), AASHTO LRFD Codes adopts the load combination for STRENGTH-V limit state as

$$1.25D + 1.35(L + I) + 0.40W \tag{7}$$

1.1.7 Statistical Resistance Models

Although the capacity (resistance) of a member is treated deterministic in the Load Factor Design (LFD), in reality there is some uncertainty associated with the material strength, member cross-sectional dimensions. Therefore, the resistance, R, is also a random variable. The cause of uncertainty can be put into three categories:

Material – strength of material, elastic modulus, cracking stress, etc.

Fabrication – geometry, dimensions, etc.

Analysis – approximate analytical method, idealized stress and strain distribution models.

Usually, the variability of the resistance of components has been quantified by tests, observations of existing bridges and by engineering judgment. Most of this information is available for the basic structural material and components such as statistical data for concrete or steel stress-strain curves, concrete or steel elastic modulus, or steel I-section hot-rolled members. However, structural members are often made of several materials such as R/C or P/S girders. Since the test data on the resistance of such composite members is not always available, it is often necessary to develop statistical resistance models using available material test data and numerical Simulation (such as Monte Carlo Simulation). The resistance model R is to consider the resistance as a product of the nominal resistance, R_n, used in design and three parameters that account for the uncertainty of material strength factor M, fabrication (dimensions) factor F, and analysis (professional) factor P.

$$R = R_n MFP \tag{1}$$

Therefore the statistical parameters of a resistance model are

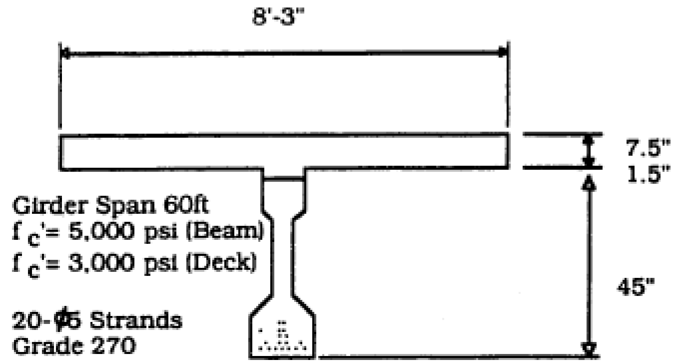
$$\mu_R = R_n \mu_M \mu_F \mu_P \tag{2}$$

and

$$V_R = \left(V_M^2 + V_F^2 + V_P^2\right)^{1/2} \tag{3}$$

where μ_M, μ_F, and μ_P are the mean values of M, F, and P respectively. V_M, V_F, and V_P are the coefficients of variation of M, F, and P, respectively. $\mu_M, \mu_F, \mu_P, V_M, V_F$, and V_P were developed for steel girders with composite and non-composite, reinforced concrete T-beams, and prestressed concrete AASHTO-type girders and are available in the literature.

A typical statistical moment capacity model of composite AASHTO type 3 P/S I-Girder is shown in Figure 1.1.7.1. The typical stress-strain curves for concrete, prestressing strand were used. In the analysis, the curves were generated by Monte Carlo Simulations. The long-term strength changes of the concrete and steel are ignored in the analysis.



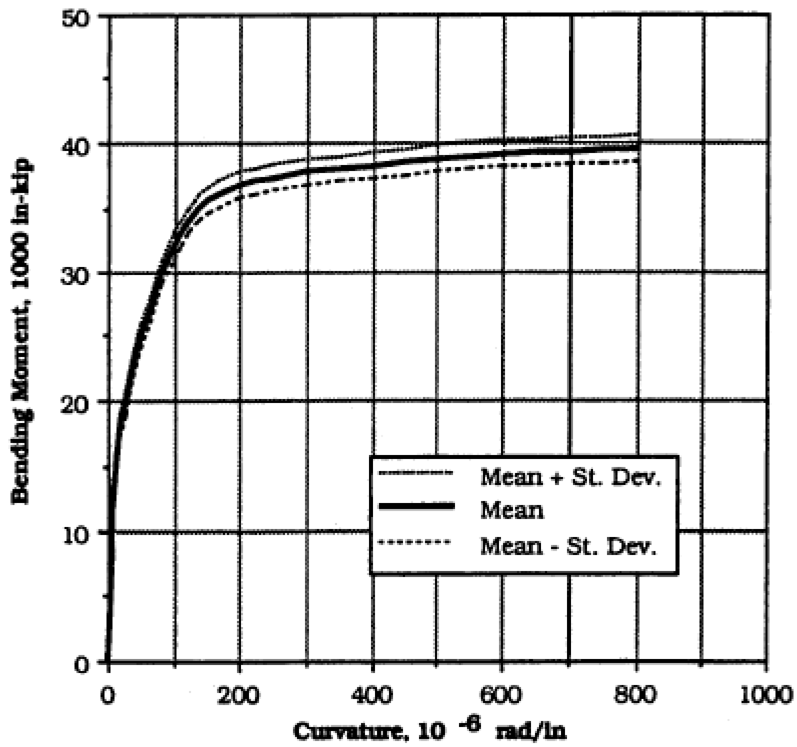


Figure 1.1.7.1 Moment – Curvature Curves for Type III AASHTO Composite Girder

The statistical parameters of resistance for steel girders, reinforced concrete T-beams and prestressed concrete girders are shown in Table 1.1.7.1.

Statistical Resistance Models

Table 1.1.7.1 Statistical Parameters Resistance

Type of Structure	FM		P		R	
	λ	V	λ	V	λ	V
Non-Composite Steel Girders						
Moment (compact)	1.095	0.075	1.02	0.06	1.12	0.10
Moment (non-com.)	1.085	0.075	1.03	0.06	1.12	0.10
Shear	1.12	0.08	1.02	0.07	1.14	0.105
Composite Steel Girders						
Moment	1.07	0.08	1.05	0.06	1.12	0.10
Shear	1.12	0.08	1.02	0.07	1.14	0.105
Reinforced Concrete						
Moment	1.12	0.12	1.02	0.06	1.14	0.13
Shear W/Steel	1.13	0.12	1.075	0.10	1.20	0.155
Shear No Steel	1.165	0.135	1.2	0.10	1.40	0.17
Prestressed Concrete						
Moment	1.04	0.045	1.01	0.06	1.05	0.075
Shear W/Steel	1.07	0.10	1.075	0.10	1.15	0.14

1.1.8 Reliability Index For Current AASHTO Standard Specification

In order to develop the LRFD codes, a target reliability index needs to be determined. To determine the target reliability index, a set of bridges were selected. The selected set includes about 200 representative existing bridges in different geographical locations in the U.S. For the selected bridges, moments and shears of girders are calculated due to dead loads and live loads. The basic design requirement according to AASHTO Standard Specification is

$$1.3D + 2.17(L + I) = \phi \ R_n \tag{1}$$

where D, L, and I are moments (or shears) due to nominal dead load, live load and impact using current AASHTO. R_n is the required resistance and ϕ is the resistance factors based on the AASHTO Standard Specifications. The required resistance, R_n, is calculated as

$$R_n = \frac{1.3D + 2.17(L + I)}{\phi} \tag{2}$$

Once R_n is calculated, the mean and standard deviation (i.e., μ_Q and σ_Q) are calculated for the total load effect based on the statistical parameters of individual load effects as shown in Table 1.1.6.2. The mean and standard deviation of resistance (i.e., μ_R and σ_R) can be obtained from R_n and Table 1.1.7.1. By knowing statistical parameters of load and resistance effects of Q and R, the reliability index, β, can be calculated according to Section 1.1.4.2. After calculating the reliability index for all the bridges in the selected bridge set. The target reliability index was selected by AASHTO to be 3.5 for moments and shears.

The reliability indices of moment and shear based on AASHTO Standard Specifications for steel girders are shown in Figures 1.1.8.1 and 1.1.8.2, respectively. Similarly, the reliability indices of moment and shear for prestressed I-Girders are shown in Figures 1.1.8.3 and 1.1.8.4, it can be seen that the reliability indices vary significantly when the girder spacing or the span length is changed. It indicates that a uniform safety level for various spans and girder spacing is not achieved if bridges are designed by the AASHTO Standard Specifications.

Reliability Index For Current AASHTO Standard Specification

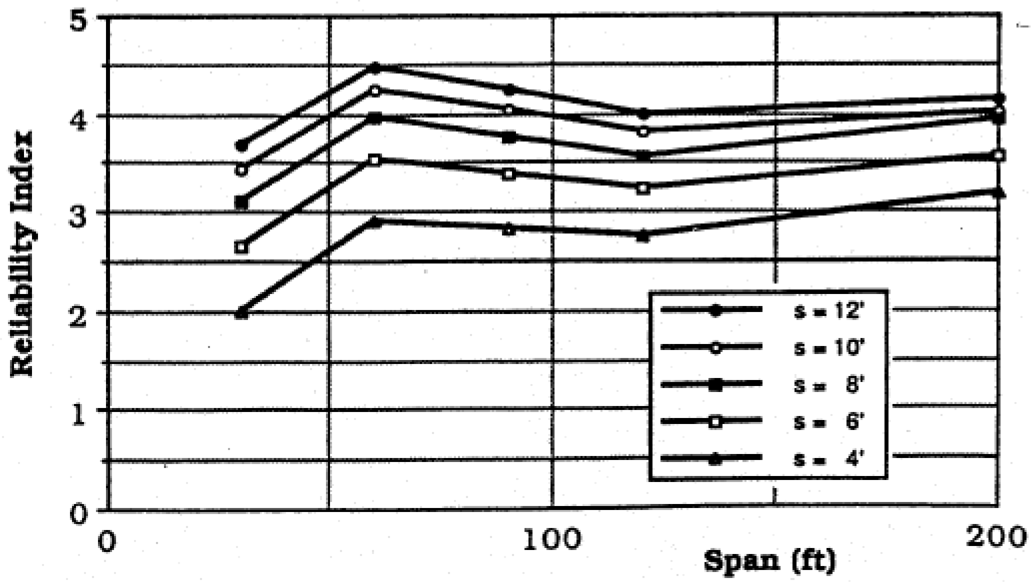


Figure 1.1.8.1 Reliability Indices for Current AASHTO; Simple Span Moment in Composite Steel Girders.

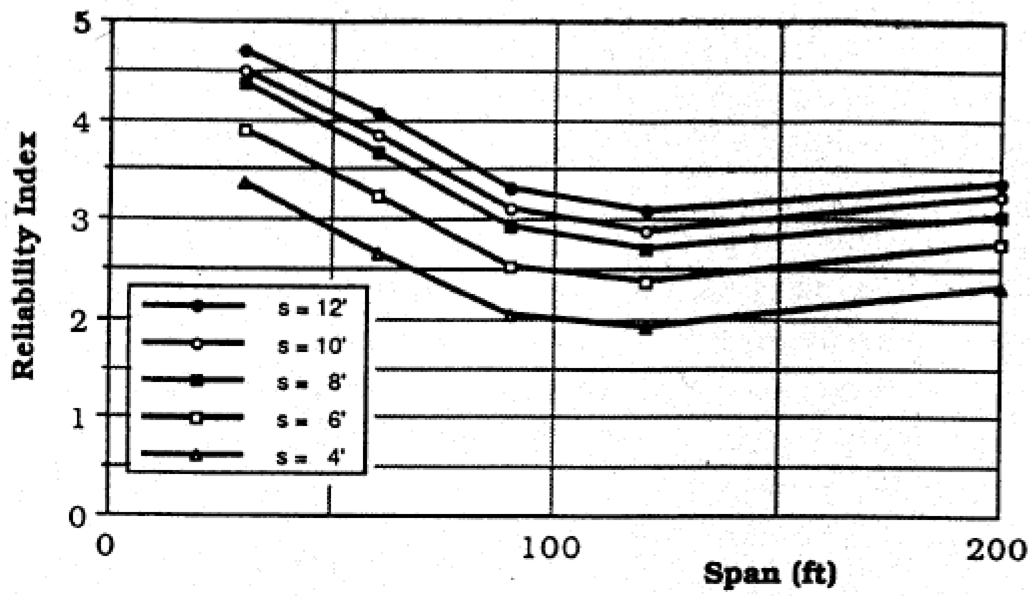


Figure 1.1.8.2 Reliability Indices for Current AASHTO; Shear in Steel Girders.

Reliability Index For Current AASHTO Standard Specification

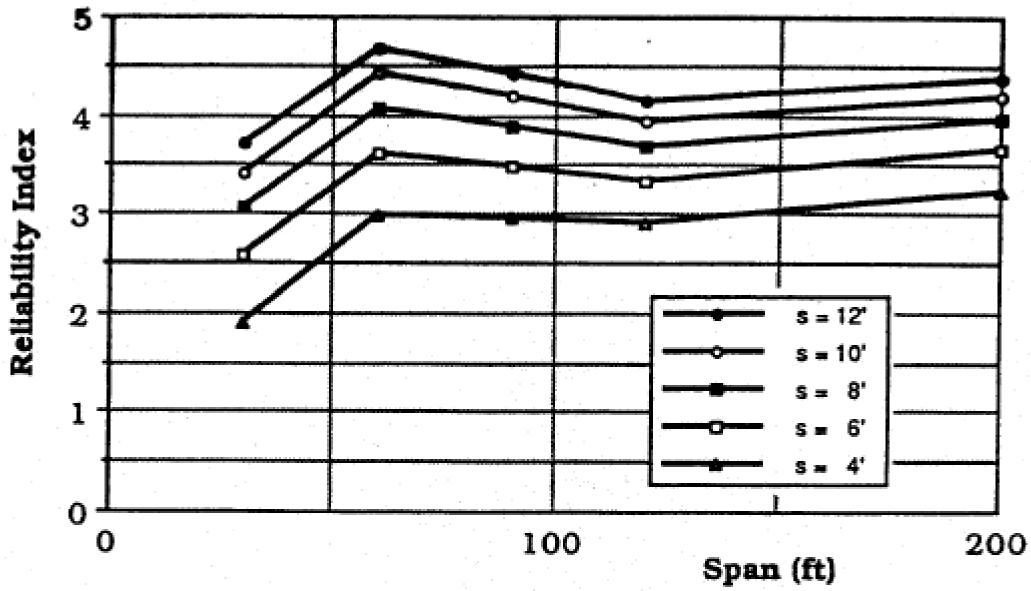


Figure 1.1.8.3 Reliability Indices for Current AASHTO; Simple Span Moment in Prestressed Concrete Girders.

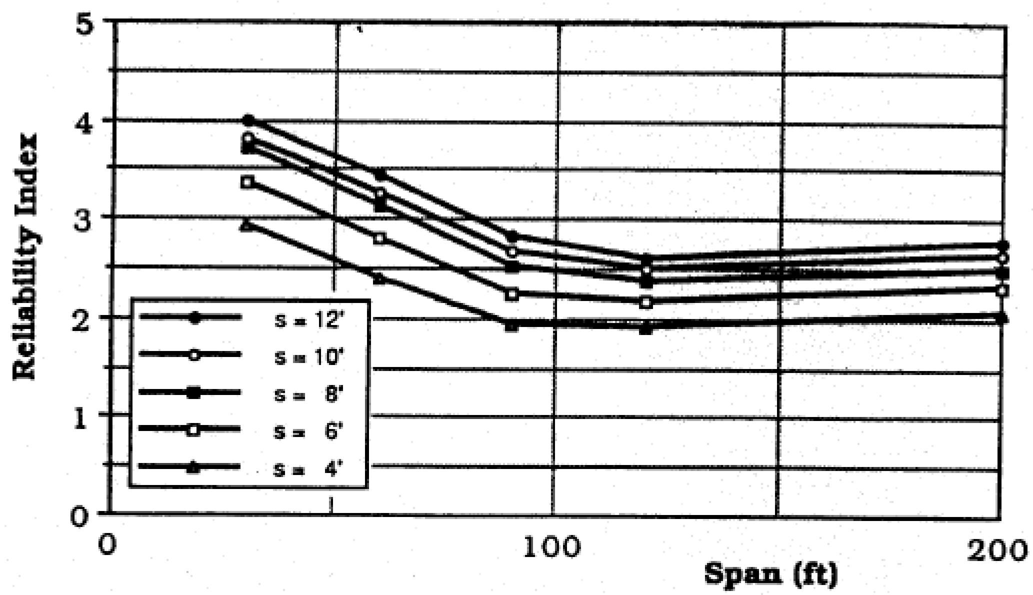


Figure 1.1.8.4 Reliability Indices for Current AASHTO; Shear in Prestressed Concrete Girders.

1.1.9 Calibration of Load and Resistance Factors

The LRFD Code objective was to minimize the discrepancy between the reliability index of designed bridges and the target index $\beta_T = 3.5$. The reliability indexes below the target value of *3.5* are generally not acceptable. The bridges chosen for the calibration include steel girder bridges (composite and non composite), reinforced concrete bridges (T beams) and prestressed concrete bridges (AASHTO girders). The span range is from *30 ft* to *200 ft* and girder spacing range is from *4 ft* to *12 ft.*

The recommended load factors of dead loads and live load described in Section 1.1.6.4 are used for the calibration. In the selection of the resistance factors, the acceptance criteria is closeness to the target value of the reliability index, $\beta_T = 3.5$. Various set of resistance factors, ϕ, are considered. Resistance factors used in the code are rounded off to the nearest 0.05. For each value of ϕ, the minimum required resistance, R_{LRFD}, is determined from the following equation,

$$R_{LRFD} = \left[1.25(D_1 + D_2) + 1.5D_3 + 1.70(L + I)\right]/\phi \qquad (1)$$

For a given resistance factor, material, span and girder spacing, a value of R_{LRFD} is calculated using equation (1). Then, for each value of R_{LRFD} and corresponding loads, the reliability index is computed based on Section 1.1.4. The calculations shown that the reliability indices for bridges designed by the LRFD code do not depend on girder spacing (i.e. the change of reliability index due to change of girder spacing is very minimum). The reliability indices of moment and shear based on LRFD Code for steel girders are shown in Figures 1.1.9.1 and 1.1.9.2 respectively. The reliability indices of moment and shear for prestressed I-Girder are shown in Figures 1.1.9.3 and 1.1.9.4, respectively. From the calibration, the recommended resistance factors for the LRFD code are shown in Table 1.1.9.1.

Table 1.1.9.1 Recommended Resistance Factors.

Material	Limit State	Resistance Factor, ϕ
Non-Composite Steel	Moment	1.00
	Shear	1.00
Composite Steel	Moment	1.00
	Shear	1.00
Reinforced Concrete	Moment	0.90
	Shear	0.90
Prestressed Concrete	Moment	1.00
	Shear	0.90

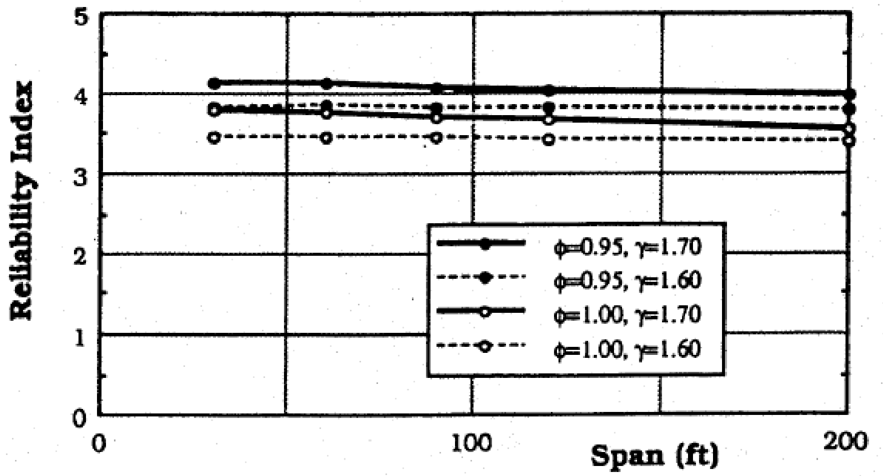


Figure 1.1.9.1 Reliability Indices for LRFD Code, Simple Span Moments in Composite Steel Girders.

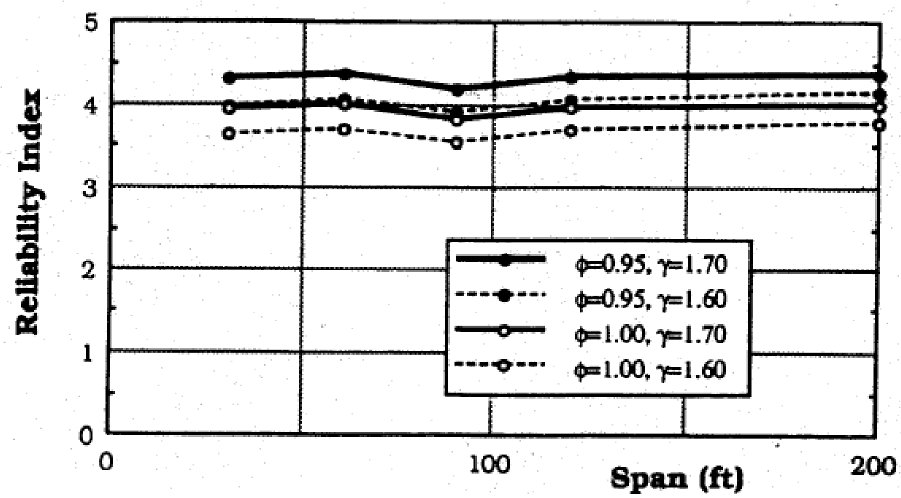


Figure 1.1.9.2 Reliability Indices for LRFD Code, Simple Span Shears in Steel Girders.

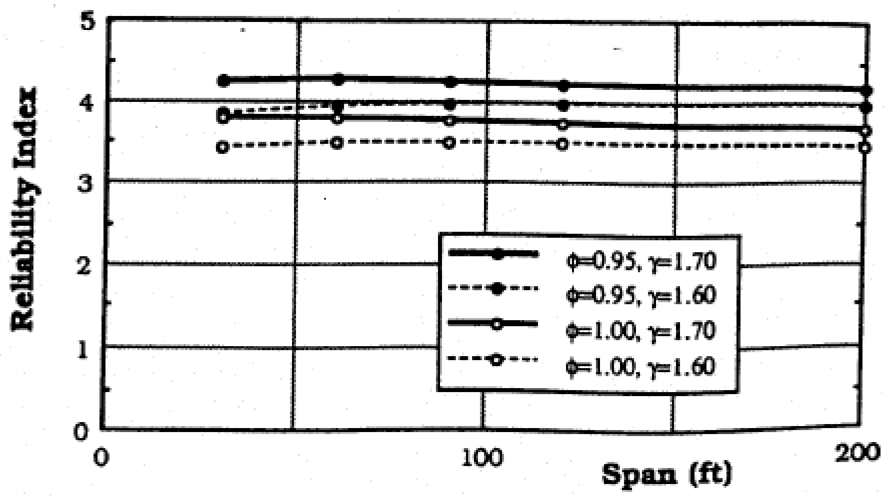


Figure 1.1.9.3 Reliability Indices for LRFD Code, Simple Span Moments in Prestressed Concrete Girders.

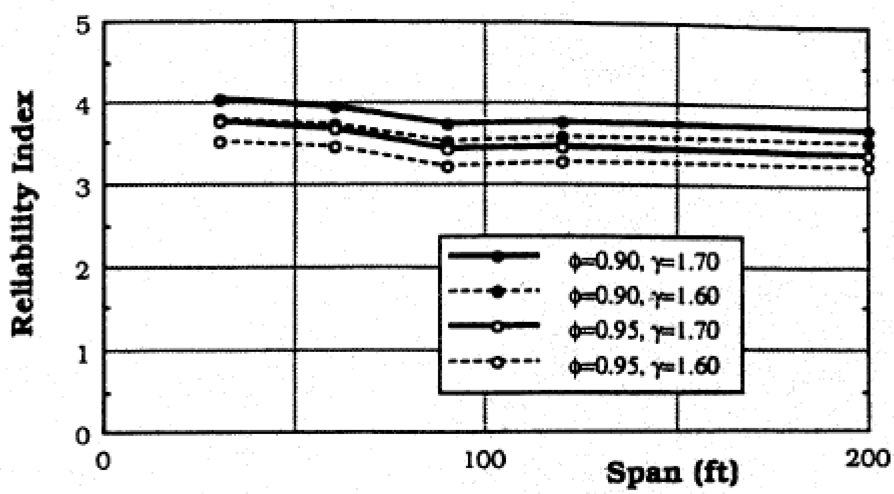


Figure 1.1.9.4 Reliability Indices for LRFD Code, Simple Span Shears in Prestressed Concrete Girders.

From Figures 1.1.9.1 through 1.1.9.4, it can be seen that the LRFD Code results in a considerably reduced scatter of β values by comparing with Figures 1.1.8.1 through 1.1.8.4, and more uniform reliability is achieved. For easier comparison with the current AASHTO Standard Specifications, a resistance ratio, r, is defined as

$$r = R_{LRFD} / R_{HS20} \quad (2)$$

Equation (2) is a measure of the actual changes of the code requirements. Value of $r > 1$ corresponds to LRFD Code being more conservative than the current AASHTO Standard Specifications, and $r < 1$ corresponds to LRFD being less conservative than the current AASHTO Standard Specification. Figures 1.1.9.5 and 1.1.9.6 shows the moment and shear resistance ratios for steel girders, respectively. Similarly, the moment and shear resistance ratios for P/S I-Girders are shown in Figures 1.1.9.7 and 1.1.9.8, respectively. In general, LRFD is less conservative for the resistance moment when girder spacing increases. It is more conservative for the resistance shear regardless of the girder spacing when span length is greater than 60' for steel girder bridges. It is also more conservative for the resistance shear regardless of the girder spacing and span length for prestressed I-girder bridges.

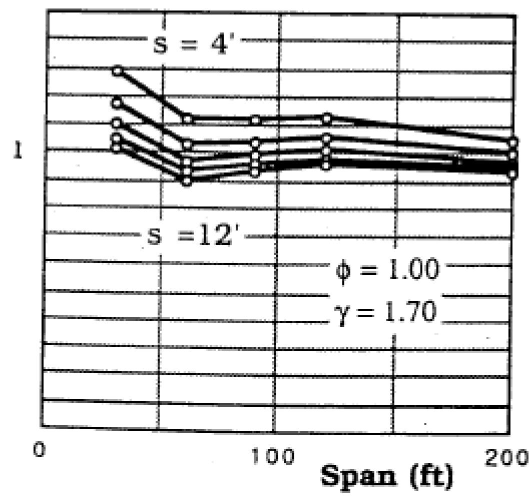


Figure 1.1.9.5 Resistance Ratios, $r = R_{LRFD}/R_{HS20}$, for Simple Span Moment, Composite Steel Girder Bridges for Girder Spacing *s = 4, 6, 8, 10, and 12 ft.*

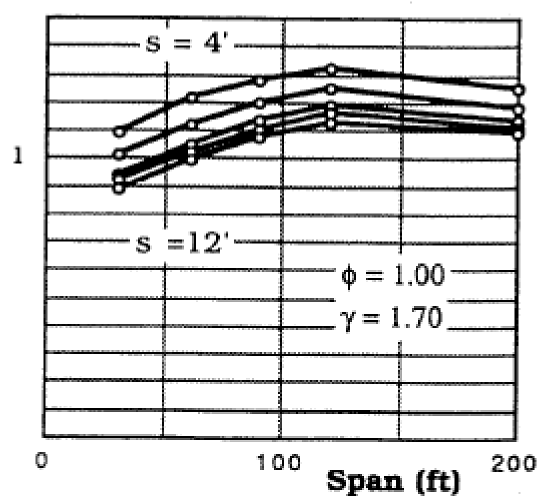


Figure 1.1.9.6 Resistance Ratios, $r = R_{LRFD}/R_{HS20}$, for Simple Span Shear, Steel Girder Bridges, for Girder Spacing *s = 4, 6, 8, 10, and 12 ft.*

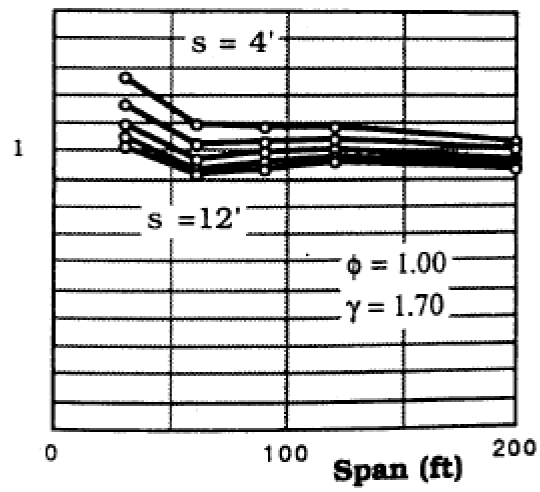


Figure 1.1.9.7 Resistance Ratios, $r = R_{LRFD}/R_{HS20}$, for Simple Span Moment, Prestressed Concrete Girder Bridges, for Girder Spacing *s = 4, 6, 8, 10, and 12 ft.*

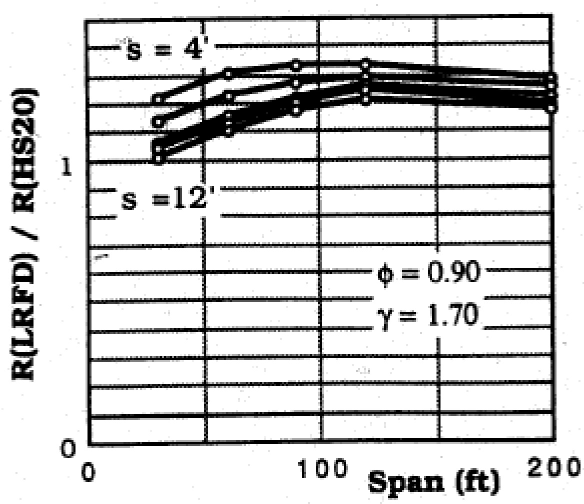


Figure 1.1.9.8 Resistance Ratios, $r = R_{LRFD}/R_{HS20}$, for Simple Span Shears, Prestressed Concrete Girder Bridges, for Girder Spacing *s = 4, 6, 8, 10, and 12 ft.*